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THE TOTAL ECLIPSE OF THE SUN, MAY 28, 1900.

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Fair weather and splendid success seem to have been the lot of most of the astronomers who were scattered all along the path of totality across the United States. Probably no eclipse has ever been so extensively and successfully observed and much is to be expected from the photographic and spectrographic results which have been obtained.

It is too early to have obtained copies of the photographs taken at the various stations, so that we can present in this number of Popular Astronomy only some of those obtained by our own party, but we shall expect to present others in due time.

The frontispiece is a reproduction of one of the photographs of the corona taken near the end of totality with the 8-inch photographic refractor belonging to Goodsell Observatory. The exposure was 5 seconds, on a Seed nonhalation plate, backed with a paste of lampblack in 6 parts of essence of cloves to 1 part of turpentine. The plate was developed with weak pyro developer. It shows the two great streamers of the corona extending toward the west and one toward the east to the edge of the field of the telescope, besides many small streamers, some within and some alongside of the large ones. The polar streamers and rifts are well defined and their curvature is quite marked. At the lower right hand edge of the Moon's disk a row of brilliant prominences may be seen, one extending one-sixteenth of the Sun's diameter, or 54,000 miles from its edge. The photographic print shows these prominences much less plainly than the negative, and in the engraving they may be expected to be almost wholly lost.

The expedition from Carleton College to observe the eclipse was made possible by the generous enthusiasm of the junior class in the college, a large part of which has been studying Young's General Astronomy during the fall and winter terms, and Campbell's Practical Astronomy during the spring term. The class voted to defray the expenses of the expedition to the extent of one hundred and fifty dollars. We are greatly indebted

also to the officials of the Chicago Great Western railway, through whose courtesy the party and instruments were conveyed to Chicago and back free of charge. Mention should also be made of the kind interest shown by the officials of the Chicago and Eastern Illinois and other roads, over which we passed, in their successful efforts to have our instruments handled carefully in transit.

The Faculty and trustees of Guilford College, near Greensboro, N. C., were much interested in the expedition and contributed largely to its success, in securing a favorable location, furnishing entertainment and rendering valuable assistance in the observations, a large party of the teachers and students being present at the observing station and taking assigned parts in the work.

The expedition was located near Southern Pines, N. C., on an eminence upon the large fruit farm of Mr. John Van Lindley, who very generously offered us the use of his commodious house and grounds, and bore the expense of our maintenance there. The Carleton party consisted of the writer as the astronomer in charge, with Professor and Mrs. A. H. Pearson, of Carleton College, as assistants. Mrs. Clements, of Faribault, also accompanied the party to North Carolina and assisted in sketching the corona. Our apparatus consisted of the 8-inch Clark photographic telescope of 9 ft. focus, the 6-inch Brashear stellar camera of 3 ft. focus and the 2½-inch Darlot lens of about 8 inches focus, all upon the same mounting and driven by clock work. With this apparatus, which was operated by the writer without assistance 8 photographs were taken: 4 of the corona during totality with the 8-inch telescope, the exposures being 1, 5, 30, and 5 seconds, the first two on Cramer crown plates and the last two on Seed nonhalation plates; 2 instantaneous exposures just a few seconds after totality upon nonhalation plates, both showing portions of the corona and chromosphere together with a reversed image of the sunlight crescent. With each of the cameras a single exposure was made, lasting 93 seconds, for the purpose of showing the outer extensions of the corona and any possible bright intra-mercurial planets. The last mentioned two photographs were both successful in showing the three long streamers of the corona extending out to at least two diameters from the edge of the Moon, but present only negative evidence of any bright planet other than Mercury near the Sun. Mercury is a conspicuous object upon both plates, but very few stars are noticeable.

The skylight was quite intense and comes out very black in the

negatives. It may be that when I get time to go over the plates systematically with a microscope I shall find many more star images than I think now are there.

The plate exposed in the 6-inch camera was an 8x10 Seed non-halation backed with essence of cloves and lamp black. The field covered was 16° in diameter. The plate exposed in the 2½-inch camera was a 4x5 Cramer slow isochromatic, covering a field of 30° diameter, 20° of which is quite sharp in definition. Both plates were developed with quite strong hydrochinon developer in order to bring out strong contrasts. Both plates give practically the same extent of coronal streamers. With the 6-inch camera the chromosphere comes out very brilliant and partially reversed, while the prominences are entirely reversed.

Another piece of apparatus used was a prismatic camera consisting of two small 60° prisms, and a 3-inch visual lens of 30 inches focus, which were mounted in a box constructed at our station, after the photographic telescope had been got into adjustment. We had not fully decided to use these but took them along in case we should find time to mount them, yet doubtful whether the prisms, 1½ inches face, would give light enough for successful photographs of the "flash" spectrum. We were urged by Professors Hale and Frost, whom it was our great pleasure to accompany from Chicago to Wadesboro, to by all means make the attempt. The result shows that the combination of these small prisms and lens was quite sufficient to give at least the chromospheric spectrum. We had no collimator along and had to depend for the focus upon the sharpness, visually, of the edge of the continuous spectrum of the Sun. We adjusted it in this way as nearly as possible for the visual spectrum from D to K and used Cramer's instantaneous isochromatic plates 5x8.

With this camera, stationary during totality, Professor Pearson, assisted by Mr. Kelly of Haverhill, Mass., made four exposures, one beginning at the instant time was called by myself for the beginning of totality, as seen through the 5-inch guiding telescope, and ending at the count of 3 seconds; another from 15 to 75 seconds for the purpose of photographing the coronal ring; a third beginning at 87^s and ending at the call of time for the close of totality; and a fourth as short as possible, at 100^s, 6 seconds after the sunlight reappeared.

The photographs show a spectrum 5 inches long extending from D_3 to a little beyond K. Unfortunately, in planning for the long exposure without driving apparatus, I placed the edge of the prisms parallel to the direction of diurnal motion, not rea-

lizing that the curved cusp-lines of the Sun and chromosphere would be very nearly perpendicular to this direction. As a result the lines cross each other, all being nearly tangent at their brightest parts to a line lengthwise of the spectrum. They are therefore not suitable for reproduction and will be difficult of measurement. The first shows 12 very prominent chromospheric lines besides about 85 lesser ones. The plate is too much fogged to show the coronium line. The second is much blurred of course by the drift during the 60 seconds exposure, but is very interesting. It shows 15 prominences around the elliptic rings corresponding to the H and K lines, each prominence being drawn out into a line by the drift. The brighter of these prominences are shown in several other lines of the spectum, notably D_3, F, G' and h. The coronal spectrum is continuous from below D_3, running off the plate above K. The spectrum of the polar regions is faint forming a dark center to the band of the continuous spectrum. In the place of "coronium" line a ring is faintly shown. I was not able to see it until today, but when once pointed out it is easily seen. There is a faint suggestion of another similar ring near H and K, a little less refrangible than H, at about wavelength 3988. It is very vague but seems to me to be of the same form as its neighbor H, except as to the prominences.

The third plate, exposed just before the close of totality was jarred so badly during exposure that it is useless, but the coronium ring is plainly shown. An exceedingly faint suggestion of a ring is shown near H, but not sufficiently definite to be used as a verification of that in the second plate.

The fourth plate exposed 6 seconds after totality came out a surprise to me. It was intended to give the dark lines in the solar cusp spectrum, but it does not show a single dark line that I can find. The west edge of the spectrum is continuous, (this may be partly due to the overlapping of the lines) while beside it is a beautiful bright line spectrum of one of the cusps, the lines most of them running into the continuous spectrum, and some of them exceeding it in brightness. I have counted 170 lines between D_3 and K and there are fragments of perhaps as many more. They are distributed as follows:

Below D_3	2	From h to H	15
From D_3 to F	37	From H to K	3
From F to G'	58	Beyond K	2
From G' to h	56		

I have been puzzled to account for the total absence of dark lines in this photograph, for several minutes before totality and

again after totality they were visible to the eye upon the ground glass of the camera. It would seem possible that we may have caught that moment when the radiations of the reversing layer and those of the reappearing photosphere were of equal intensity so that no reversal would be produced.

The party from Guilford College, associated with us, consisted of Professor George W. White and Mrs. White, Professor Robert W. Wilson, Professor Hodgins, Miss Field and students Walter Hobbs, Thomas Hinton, Homer Ragan, Lacy L. Barbee and Emmit Shepard. Professor White photographed the corona with a 3-inch visual telescope, exposing four plates for 1 and 2 seconds each during totality. He obtained four very good photographs of the middle parts of the corona. Professor Wilson counted off the seconds of time during totality, so that those making the photographic exposures might follow their assigned programs without looking at timepieces. Messrs. Barbee and Shepard noted on a sidereal chronometer and an ordinary watch independently the times of the contacts as observed by the writer with the 5-inch telescope. Professor Hodgins, Mrs. White, Miss Field and Mrs. Couts, under the direction of Mrs. Pearson, sketched portions of the corona, using white crayon on blue paper. Their results agree well with the photographs in the main features of the corona. Four of the students watched for the shadow bands and noted their direction.

Description of the Eclipse.

For a popular description of the eclipse the following abstract from an article in the Haverhill (Mass.) Evening Gazette, by Mr. Austin P. Nichols, who was present with our party during the eclipse, is as good as any I have seen:

"As the Moon crept further and further over the face of the Sun the interest and excitement increased. Several times each member of the party took their assigned places and while the astronomer in charge of the chronometer counted off the 94 seconds of expected totality, rehearsed the part they were expected to perform an hour later. The light gradually diminished and the air became somewhat cooler, but the change was not especially marked. The writer took his place on the roof of a barn, with instructions to note the approach of the shadow from the westward. Another member of our party had been impressed into the service of Professor Pearson to assist him in photographing the spectrum. The martyr to science who was to count the seconds, and was therefore debarred from seeing the eclipse, took charge

of the chronometer. A bewildered rooster gave a feeble, inquisitive crow, and all was in readiness for the great event.

"Five minutes, called the timekeeper, and everyone busied themselves to be sure that everything was all in order at the last moment—four minutes, three, two, one, and a great hush fell over the party like that which occurs between a lightning flash and the thunder clap. The sky rapidly darkened, except that around the horizon were tints of orange, and to the southwestward the country was wrapped in the gloom of the approaching shadow. Time! called the man at the telescope, and the eclipse was upon us.

"In an instant everything was transformed. The darkness was not intense, it was about that of late twilight, and the hands of a watch could easily be made out. Everyone was busy at their appointed task, not a sound was heard but the calling of the seconds by the timekeeper. Only 94 of them, and so much must be done in that brief time. One fairly begrudged every second as it was called off.

"Up in the heavens where an instant before had been the brilliant crescent of the Sun, hung the black disk of the Moon; around its edge was a narrow but brilliant ring of an indescribably beautiful pearly, silvery shimmering light—the inner corona. Extending out into space on either side of the Sun, like the wings of a gigantic bird, were the streamers of the outer corona less brilliant than the inner ring, but bright and equally beautiful. The light of this outer corona was not uniform but varied in intensity, giving the appearance of structure to it; the writer particularly noticed a leaf-like form in one of the wings, similar to those observed in previous eclipses. The outer corona wings were strikingly similar to the tail of a brilliant comet, in appearance, although nothing has yet been observed to indicate that they have the same composition. The length of these wings seemed to be from three to four times the diameter of the Sun, although to other observers they appeared much shorter. A few red prominences, due to masses of red hot hydrogen gas thousands of miles high, were also observed around the edge of the sun, but to the unaided eye they were not very conspicuous, and could only be seen to good advantage in a telescope. Close to the Sun the planet Mercury shone with an unaccustomed brilliancy, while lower down in the east Venus, the evening star, glowed even more brightly. No other stars were noticed.

"A more beautiful or more impressive scene is rarely witnessed, but the time was all too short. Probably no one who observed

the eclipse but felt that he had left something undone that he ought to have done, but one cannot do everything in a minute and a half.

"The monotonous counting of seconds continued—ninety-one, two, three, four. A flash of sunlight like the sudden lifting of a thunder cloud, the corona vanished like a burnt-out firework, familiar features of the landscape shone out again, and, except for a certain pale, ghastly light, which lasted for nearly half an hour, everything took on its usual appearance. Everyone heaved a sigh of relief, and the great eclipse of 1900 was a thing of the past."

THE CORONA.

The general form of the corona, as it appeared to me in the 20 seconds I had in which to look, while the 30 second exposure was going on, struck me as being very similar to that which I had seen in 1889; the difference being that it was reversed east and west, the two equal parallel streamers extending toward the west instead of east and the long single streamer with smaller attendants extending toward the east. Thus the view is confirmed which has been tentatively held for some years that certain types of corona are associated with the phases of the sunspot period, and that the type which corresponds to the sunspot minimum is characterized by great streamers parallel to the Sun's equator and by very marked polar filaments and rifts.

The four photographs taken by the writer with the 8-inch telescope do not show any noticeable changes in the structure of the corona in the one minute and a half. They differ from each other because of differences of exposure and of development. Comparing any one of them with Barnard's print of the corona of Jan. 2, 1889, one would almost say it is the same corona rotated half way round on the polar axis.

THE SHADOW BANDS.

These were seen three minutes before totality and for an equal length of time after totality. A white sheet was stretched upon the ground for these observations, but it was unnecessary for the bands could be seen all around us upon the light colored sand. They came approximately from the direction from which the great shadow approached, but the direction of movement was not constant and at one time it seemed to be from the northwest. The lines of the shadows were not smooth curves, but were sinuous and continually wavering. I had asked the young men,

if possible, to count the number of bands seen upon the sheet at once and how many passed in 10 seconds, but the movement was too swift for this and they could only catch the direction of the wave fronts by laying sticks down parallel to the bands. This they did four times, 3 minutes before totality; just as the great shadow came; just after it receded; and when they last saw the bands distinctly. The following angles with the meridian were measured by myself a few minutes after totality was over:

I S 42.°0 E. II S 37.7 E. III S 52.°4 E. IV S 37.7 E.

According to the map of the path of totality the direction of movement of the shadow was about N 60° E at Southern Pines.

Various estimates of the distance between the successive shadow bands placed it at from 6 to 8 inches.

OBSERVATIONS OF CONTACTS.

The four contacts were observed by the writer, the first three with the 5-inch guiding telescope of 9 feet focal length, the last with the 3-inch telescope belonging to Guilford College. The sidereal chronometer Bond 374 was compared by telephone with the signals transmitted from Washington to Pinehurst, N. C., on May 26 and 27 and with the mean time chronometer Negus 1749 at the U. S. Naval Observatory Station at Pinehurst on the afternoon of May 28, giving consistent rates between times. The watch used had a variable rate and the times noted on it are only rough checks of the chronometer times. The position of our station was determined by a rough triangulation from the station at Pinehurst and cannot be determined very exactly until certain data are obtained from that station, but the position was very nearly in longitude 79° 24′ and latitude 35° 12′. The predicted times calculated with these data before the eclipse are here given for the sake of comparison, for the observed duration of totality agreed exactly with computed duration, which does not appear to have been the case at all the stations.

	Bond 374.	Corrections.	Reduced Greenwich Time.	Watch (corrected) Central Standard Time.	Computed Greenwich Time.
	h m s	m s	h m s	h m s	h m s
First contact....	22 57 19.5	+ 43 48.6	0 36 45.1	6 36 41	0 36 48.2
Second contact.	0 07 02.0	+ 43 48 8	1 46 16.4	7 46 14	1 46 26 5
Third contact...	0 08 36.0	+ 43 48.8	1 47 50 1	7 47 48	1 48 00 6
Fourth contact.	1 28 01.7	+ 43 48.9	3 07 02.9		3 07 08.3

AT OTHER STATIONS.

Pinehurst, N. C.—As the United States Naval Observatory station at Pinehurst was only 3 miles to the west of our loca-

PLATE XII.

THE CARLETON AND GUILFORD COLLEGE OBSERVING PARTIES

At their Station near Southern Pines, N. C., May 28, 1900.

POPULAR ASTRONOMY, No. 76.

tion, we had the opportunity of visiting with the astronomers and physicists there, seeing the apparatus, and becoming pretty well acquainted with the work which was to be done. The station was under the charge of Professor A. N. Skinner, who determined the latitude and longitude and had general oversight of all the work.

The most noticeable object as one approached the station was a large scaffold supporting the objective of the 40 foot telescope with which Mr. A. L. Colton was to secure large photographs of the corona, by the method which was so successfully used by Professor Campbell of the Lick Observatory in the Indian eclipse of 1898. The objective was of only 5 inches aperture but of 40 feet focal length. The tube was made of white canvas, supported by a skeleton framework of steel tubing. The plate-holder was placed in a pit covered by a dark hut, and was moved by a clepsydra during the short interval of totality. Mr. Colton was to expose five 14 x 17 Seed nonhalation plates, the exposures being 2^s, 10^s, 45^s, 10^s and 2^s. He was assisted by Mr. M. G. Skinner.

A long shed with one-half of the roof covered with canvas, which could be rolled up out of the way, sheltered a row of equatorial telescopes, polariscopes and cameras which looked formidable to the spectator. The most important work, however, as it seemed to me was to be done with some instruments under a rougher looking shed partly covered only by loose boards. Here were three great slitless spectroscopes: one, operated by Dr. J. S. Ames of the Johns Hopkins University, having a 6-inch concave grating with 15,000 lines to the inch and giving a spectrum 2 feet long; another, also under Dr. Ames' charge, having a flat grating of the same dimensions as the concave grating; and a prismatic camera, with a 60° prism whose faces were 6 inches square and a lens of about 6 feet focal length. The last was under the charge of Dr. Chase of Yale University. Dr. Ames was assisted by Messrs. Hough, Rees and Gilbert of Johns Hopkins University. I understand that Dr. Ames made 15 exposures with the concave grating spectrograph and it is to be hoped that these have all turned out well.

The battery of cameras upon a long polar axis was managed by Professor W. S. Eichelberger and Assistant Astronomer T. I. King of the Naval Observatory assisted by Messrs. Beal, Kent and Gore. One of the cameras was provided with a color screen which would allow only the green rays to pass through it, the hope being to obtain an image of the corona largely from the light due to coronium.

Professor Edgar Frisby of the Naval Observatory and Mr. Everett I. Yowell of the Cincinnati Observatory observed the contacts and made visual observations of the corona with two 5-inch equatorials. Mr. N. E. Parsons and Dr. Dorsey of the Johns Hopkins University operated two polariscopes, the former a photographic and the latter a visual instrument.

Mr. R. W. Wood of the University of Wisconsin made interesting observations of the shadow bands with a peculiar instrument called the strobo-scope. His conclusion was that the shadow bands were not periodic. He also observed with a polariscope and made an excellent drawing of the corona.

From Professor Skinner we learned that the U. S. Naval Observatory had sent out three other parties, one at Barnesville, Ga., under Professor Brown, one at Winnsboro, S. C., under Professor Ormond Stone, Director of the Leander McCormick Observatory and one at Griffin, Ga., under Professor L. E. Jewell, of the Johns Hopkins University.

Barnesville, Ga.—The instruments used here were a 40 ft. telescope and polar axis with several cameras, similar to those at Pinehurst. Professors Updegraff and See of the Naval Observatory were assisted by a number of volunteer observers.

Winnsboro, S. C.—Professor Stone assisted by volunteers operated a third 40 foot telescope for the purpose of securing large photographs of the corona upon 14 x 17 plates.

Griffin, Ga.—This station was chosen near the edge of the path of totality for the purpose of obtaining long duration of the "flash" and chromospheric spectra, the whole expedition being spectroscopic. Professor Henry Crew, of the Northwestern University, worked with a flat grating. We are sorry to learn from a recent newspaper note, that his photographs all proved failures from insufficient exposure. Professor Humphrey, of the University of Virginia, operated with a concave grating of 21 feet focal length. Professor Mitchell, of the Columbia University, was also to be located at this station.

Wadesboro.—On our return journey Professors Hale and Frost gave us an interesting account of the work at this city, which seems to have been the favorite location for observing the eclipse, chosen no doubt on account of the low percentage of cloudy weather given in the special report of the Weather Bureau. Three important expeditions were located here, from the Smithsonian Institution, Princeton University, and the Yerkes Observatory. We can only speak definitely of the work done by the Yerkes party.

Professor E. E. Barnard assisted by Mr. Ritchey made 7 exposures with a 6-inch lens of 62 feet focus mounted horizontally. The light was thrown into the telescope by a coelostat with a 12-inch mirror made by Mr. Ritchey. The exposures made varied from one-half second to thirty seconds in duration, three upon 14 x 17 plates and four upon 25 x 30 plates, the largest that have ever been used for photographing the corona, with exception of those used at the same station by the Smithsonian Institution. A newspaper report, June 14, states that one of these plates, the last exposed, duration 1 second, had been developed and brought out many interesting details close to the edge of the moon. Mr. Barnard's hope is to bring out upon the longer exposed negatives the details of the outer corona as well as the inner upon that magnificent scale, which makes the diameter of moon's disk 7 inches; and we may be sure that Mr. Barnard will do it if any man can.

Several smaller cameras, from a 6-inch Voightlander down, were operated by volunteers. Some were mounted equatorially and driven by clockwork, others were simply fixed upon posts.

Professor E. B. Frost assisted by Dr. Isham of Chicago photographed the spectrum, using three slitless spectroscopes, one with a train of three prisms, one with a single prism and one with a concave grating. The signal for photographing the "flash" spectrum was given by Professor Frost observing with a flat grating. He said that he saw only dozens of lines reverse where he expected to see hundreds. Elsewhere also the "flash" appears to have been disappointing. At Pinehurst the whole party depended upon the observation of this phenomenon for the signal for the beginning of their program during totality. The one deputed to make this observation and give the signal failed to see the "flash" and as a result several seconds of totality had elapsed before the signal was given.

The coronium line too appears to have been weaker than usual, and Professor Young who made the determination of the position of this one line his special work failed to see it.

Professor Hale assisted by Mr. Ferdinand Ellerman undertook the very delicate operation of measuring the heat of the corona with bolometric apparatus, in connection with a large siderostat kindly loaned by the Smithsonian Institution. He had a very unfortunate experience. On arriving at Wadesboro ten days before the eclipse he found that a very delicate part of the apparatus had been broken. He had been warned by previous experience and had shipped a lathe and all necessary tools as a part of the

equipment of the expedition. With characteristic energy he set about constructing a new bolometer and succeeding in making a better one than he had before and performed satisfactory preliminary experiments. All was in perfect adjustment and ready for use during totality, when just as the important moment arrived a small stick, which was used for some purpose in the dark bolometer room and had been leaned against the wall, fell and threw the galvanometer needle out of balance. The operation of balancing requires usually from two to three minutes. Professor Hale said that he never worked harder in his life than during the next minute and a half, and he succeeded in getting the instrument into balance, but only to see the sunlight reappear before any measures could be taken.

Mr. C. G. Abbott of the Smithsonian Institution by similar methods succeeded in detecting a slight amount of radiation from the corona as compared with that from the black disk of the moon. Professor Hale is confident from the results of his preliminary experiments that by these methods he can detect the change of heat at the edges of the great coronal streamers in full sunlight.

Professor A. S. Flint of the Washburn Observatory made the observations for time and position at Wadesboro, and noted the times of contact and counted the seconds of totality for Professor Frost.

The longest focus telescope ever used in observing an eclipse is probably that used by the Smithsonian Institution at Wadesboro, which had a focal length of 135 feet. This would give an image of the moon nearly 15 inches in diameter.

The telescope was mounted horizontally, the light being furnished by a coelostat. In the Chicago Journal of June 15, we find this statement concerning the photographs taken with this instrument:

"Mr. Smillie exposed six 30 x 30 plates during totality, with times ranging from one-half second to sixteen seconds. All these negatives have not yet been developed. Those of one-half second, two seconds and four seconds exposure have been hurriedly examined, however, and they give clear indications of the crossing and recrossing of filaments like the appearance of a field of grain bending in the wind. The prominences and polar streamers appear in imposing magnitude and detail."

GENERAL PERTURBATIONS AND THE PERTURBATIVE FUNCTION.

J. MORRISON, M. A., M. D., PH. D.

FOR POPULAR ASTRONOMY.

It was first proved by Sir Isaac Newton in his immortal work The Principia, that when a particle moves around a centre of force which varies directly as the mass and inversely as the square of the distance, the path or orbit described is a conic section with the centre of force in the focus, and the radius vector describes equal areas in equal times. In the case of a planet moving around the Sun, the orbit is an ellipse with the Sun in one of the foci. It is evident that the planet will move most rapidly in perihelion and most slowly in aphelion. The mean motion or that which it would have, if it described a circular orbit, is evidently equal to 360° or 2π divided by the periodic time T or $\frac{2\pi}{T}$. From perihelion to aphelion the true place of the planet will be in advance of the mean place; at aphelion the mean and true places will coincide, and from aphelion to perihelion the mean place will be in advance of the true until perihelion is reached when they again coincide and so on.

The angular distance between the true and mean places or to express it more technically, between the true and mean anomalies, is called the elliptic inequality or 'the equation of the centre,' and it is the only correction to be applied to the mean to obtain the true anomaly in the case we are now considering.

If however we suppose another planet to be added to the system, the circumstances of the motion of both planets, become much more complicated; each disturbs the motion of the other; the equable description of areas which obtained in the case of a single planet now no longer exists, and the computation of the true place of either planet is a work of prodigious difficulty. It is the famous "problem of three bodies" which has severely taxed the ingenuity and analytical skill of mathematicians since the discovery of the law of universal gravitation.

In this and subsequent papers we purpose to develop as clearly and as briefly as the difficulties of the problem will admit, the formulae for undisturbed and disturbed motion, so as to enable the reader to understand the more abstruse and elaborate developments of LaPlace, LeVerrier and others.

Let x, y, z be the coördinates of a planet referred to the centre

of gravity of the Sun S, as the origin and r its radius vector, also let m denote the ratio of the mass of the planet (m) to that of the Sun or $m = \frac{\text{mass of planet}}{\text{mass of Sun}} = \frac{\text{mass of planet}}{k^2}$ where k^2 is the well known Gaussian constant of solar attraction whose value will be determined farther on, then the mass of the planet $= mk^2$.

Let x', y', z' be the coördinates, r' the radius vector and $m'k^2$ the mass of a second planet (m') and similarly for other bodies of the system and let ρ, ρ_1, ρ_2 be their mutual distance, or mm', mm'' etc., then we shall have

$$\begin{aligned} r^2 &= Sm^2 = x^2 + y^2 + z^2 \\ r'^2 &= Sm'^2 = x'^2 + y'^2 + z'^2 \\ r''^2 &= Sm'' = x''^2 + y''^2 + z''^2, \text{ etc.} \qquad (1) \\ \text{and } \rho^2 &= mm'^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2 \\ \rho_1^2 &= mm''^2 = (x'' - x)^2 + (y'' - y)^2 + (z'' - z)^2, \text{ etc.} \end{aligned}$$

Considering only three bodies, the Sun S, the disturbed planet (m) and the disturbing planet (m') and putting for the sake of brevity $k^2 + k^2m$ or $k^2(1 + m) = \mu$, it is evident that in the relative motion of (m) around S, it will be acted on by the three forces $-\frac{\mu}{r^2}$, $\frac{k^2m'}{\rho^2}$ and $-\frac{k^2m'}{r'^2}$ respectively directed along the lines mS, mm' and $m'S$, and since the cosines of the angles which the directions of each of these forces make with the axis of x, are respectively

$$\frac{x}{r}, \quad \frac{x' - x}{\rho} \text{ and } \frac{x'}{r'}$$

We shall have for the components of these forces parellel to the same axis

$$-\frac{\mu x}{r^3}, \quad \frac{k^2m'(x' - x)}{\rho^3} \text{ and } -\frac{k^2m'x'}{r'^3} \text{ respectively,}$$

the first and third being negative because the force tends to diminish the coördinates x, y and z.

For the components of these same forces parallel to the axis of Y we shall have in a similar manner

$$-\frac{\mu y}{r^3}, \quad \frac{k^2m'(y' - y)}{\rho^3} \text{ and } -\frac{k^2m'y'}{r_1^2}$$

and for those parallel to the axis of Z

$$-\frac{\mu z}{r^3}, \quad \frac{k^2m'(z' - z)}{\rho^3} \text{ and } -\frac{k^2m'z'}{r^3}.$$

The sum of these three components for each direction is evi-

dently the total force parallel to this direction, which acts on the planet (m) and it must be equal to the accelerations $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2z}{dt^2}$ respectively. If then we extend these results to a number of bodies m'', m''', etc., we shall have for the equations of the relative motion of (m) around S.

$$\begin{aligned}
\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} &= k^2 \sum m' \left(\frac{x'-r}{\rho^3} - \frac{x'}{r'^3}\right) \\
\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} &= k^2 \sum m' \left(\frac{z'-z}{\rho^3} - \frac{z'}{r'^3}\right) \qquad (2) \\
\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} &= k^2 \sum m' \left(\frac{z'-z}{\rho^3} - \frac{z'}{r'^3}\right)
\end{aligned}$$

where the symbol $\sum$ indicates that each mass k^2m'', k^2m''', introduces a term similar to that which results from the action of m' on m — a term which we obtain by simply changing in the second members m', ρ, x' and r' into m'', ρ', x'' and r'' respectively, and so on.

To facilitate the solution of the problem, the second members of (2) are put into the following form.

From (1) we obtain

$$\frac{d}{dx}\cdot\frac{1}{\rho} = \frac{x'-x}{\rho^3}, \qquad \frac{d}{dy}\cdot\frac{1}{\rho} = \frac{y'-y}{\rho^3}, \qquad \frac{d}{dz}\cdot\frac{1}{\rho} = \frac{z'-z}{\rho^3}$$

and the terms $\frac{x'}{r'^3}$, $\frac{y'}{r'^3}$, $\frac{z'}{r'^3}$ are derivatives of $\frac{xx'+yy'+zz'}{r'^3}$ in regard to the variables x, y and z. If then we put

$$\Theta = \frac{m'}{1+m}\left(\frac{1}{\rho} - \frac{xx'+yy'+zz'}{r'^3}\right) + \frac{m''}{1+m}\left(\frac{xx''+yy''+zz''}{r''^3}\right) + \text{etc.} \qquad (3)$$

we shall have for the partial differential coefficients with respect to x, y and z

$$\begin{aligned}
(1+m)\left(\frac{d\Theta}{dx}\right) &= \sum m' \left(\frac{x'-x}{\rho^3} - \frac{x'}{r'^3}\right) \\
(1+m)\left(\frac{d\Theta}{dy}\right) &= \sum m' \left(\frac{y'-y}{\rho^3} - \frac{y'}{r'^3}\right) \\
(1+m)\left(\frac{d\Theta}{dz}\right) &= \sum m' \left(\frac{z'-z}{\rho^3} - \frac{z'}{r'^3}\right)
\end{aligned}$$

Substituting in (2) we have

$$\begin{aligned}
\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} &= k^2 (1 + m) \left(\frac{d\Theta}{dx}\right) \\
\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} &= k^2 (1 + m) \left(\frac{d\Theta}{dy}\right) \\
\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} &= k^2 (1 + m) \left(\frac{d\Theta}{dz}\right)
\end{aligned} \qquad (4)$$

which are the differential equations of disturbed motion. The force whose components are expressed by the second members of (4) is the disturbing force. It expresses the difference between the action of the bodies m', m'', &c , on m and on the Sun, resolved parallel to the coördinate axes, and the quantity Θ from which the second members of (4) are derived is called the Perturbative Function, the development of which into an infinite series, constitutes the chief difficulty in the solution of this problem. The integration of the equations in group (4) has never been effected except by a series of approximations.

We must first consider the case of a single planet moving round the Sun, subject only to the reciprocal action of the two bodies.

This is the case of *undisturbed* motion; the disturbing force will then be zero and group (4) becomes

$$\begin{aligned}
\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} &= 0 \\
\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} &= 0 \\
\frac{d^2z}{dt^2} + \frac{\mu z}{r^3} &= 0
\end{aligned} \qquad (5)$$

which are the differential equations of undisturbed motion and their integration will introduce six arbitrary constants which determine the circumstances of the orbit which the planet describes round the Sun under their mutual influence. The integration of (5) presents no great difficulty and is given in most works on Physical or Theoretical Astronomy, but as the student may not have any such works by him and in order to render these short papers as complete as practicable, we will give the integration here.

Multiply the first of (5) by y and the second by x and subtract the second product from the first, integrate the result and we have

$$\frac{zdy}{dt} - \frac{ydx}{dt} = h \qquad \text{(a constant)}$$

similarly $$\frac{zdx}{dt} - \frac{xdz}{dt} = h' \qquad \text{“} \qquad (6)$$

and $$\frac{ydz}{dt} - \frac{zdy}{dt} = h'' \qquad \text{“}$$

If we multiply these by z, y and x respectively and add the products we obtain

$$hz + h'y + h''x = 0 \qquad (7)$$

which is the equation of a plane passing through the origin of coördinates, that is, through the center of the Sun.

Let dv be the angle described by the radius vector r in the time dt, then the area described by this radius on the plane of the orbit will be $\frac{1}{2} r^2 dv$, and if i, i_1 and i_2 denote the angles which the orbit makes with the coördinate planes, xy, xz and yz respectively we shall have

$$\begin{aligned} r^2 dv \cos i &= h\, dt \\ r^2 dv \cos i_1 &= h'\, dt \\ r^2 dv \cos i_2 &= h''\, dt \end{aligned} \qquad (8)$$

because the first members of (6) are the projections of double the area described by the radius vector during the instant dt, on the planes of xy, xz and zy.

Squaring, adding and reducing we have

$$\begin{aligned} r^2\, dv &= (h^2 + h'^2 + h''^2)\, dt \\ &= H\, dt \end{aligned}$$

or $$r^2 \frac{dv}{dt} = H \qquad (9)$$

where for brevity we put $H = h^2 + h'^2 + h''^2$.

Let Ω denote the angle which the common intersection of the plane of reference and the plane of the orbit, makes with the axis of x, or the longitude of the node, then by spherical trigonometry we shall have

$$\cos i_1 = -\sin i \cos \Omega \text{ and } \cos i_2 = \sin i \sin \Omega$$

which combined with (8) and (9) give

$$\begin{aligned} h &= H \cos i \\ h' &= -H \sin i \cos \Omega \\ h'' &= H \sin i \sin \Omega \end{aligned} \qquad (10)$$

which determine i and Ω when the arbitrary constants h, h' and h'' are known, thus they give

$$\tan \Omega = -\frac{h''}{h'} \text{ and } \tan i = \frac{\sqrt{(h'^2 + h''^2)}}{h}. \qquad (11)$$

If we multiply equations (5) by $2dx$, $2dy$ and $2dz$ respectively, add and integrate we shall find

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} + 2\mu \int \frac{xdx + ydy + zdz}{r^3} = 0$$

and from the first of (1) we have

$$rdr = xdx + ydy + zdz$$

which substituted in the above and completing the integration, gives

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} - \frac{2\mu}{r} - C = 0 \qquad (12)$$

where C is an arbitrary constant.

But $dx^2 + dy^2 + dz^2$ is the square of the space described by the body in the time dt, and therefore in polar coördinates.

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} = \frac{dr^2 + r^2 dv^2}{dt^2}$$

which substituted in (12) becomes

$$\frac{dr^2}{dt^2} + \frac{r^2 dv^2}{dt^2} - \frac{2\mu}{r} - C = 0 \qquad (13)$$

Eliminating dt between (9) and (13) we obtain

$$\frac{dr}{dv} = \frac{r\sqrt{C r^2 + 2\mu r - H^2}}{H} \qquad (14)$$

But at the extremities of the major axis we must have $\frac{dr}{dv} = 0$, and therefore (14) enables us to determine the maximum and minimum values of r which we know to be $a(1+e)$ and $a(1-e)$ respectively. Equating the second member to zero and reducing we find

$$Cr^2 + 2\mu r - H^2 = 0$$

By the theory of quadratics, the sum of the two roots is equal to $-\frac{2\mu}{C}$ and their product to $-\frac{H^2}{C}$, we therefore have

$$2a = -\frac{2\mu}{C} \text{ and } a^2(1 - e^2) = -\frac{H^2}{C}$$

whence $\qquad C = -\frac{\mu}{a}$ and $H = \sqrt{a\mu(1 - e^2)}$

These values of C and H being substituted in (14) we have

$$dv = \frac{dr}{r} \frac{\sqrt{a\mu\,(1-e^2)}}{\sqrt{-\frac{\mu}{a}r^2 + 2\mu r - a\mu\,(1-e^2)}}$$

$$= \frac{dr}{r^2} \frac{\sqrt{a\mu\,(1-e^2)}}{\sqrt{-\frac{\mu}{a} + \frac{2\mu}{r} - \frac{a\mu\,(1-e^2)}{r^2}}}$$

and dividing numerator and denominator by $\sqrt{a\mu\,(1-e^2)}$ and writing $-d\frac{1}{r}$ for $\frac{dr}{r^2}$ we have

$$dv = \frac{-d\frac{1}{r}}{\sqrt{-\frac{1}{a^2(1-e^2)} + \frac{2}{ar(1-e^2)} - \frac{1}{r^2}}}$$

$$= \frac{-d\frac{1}{r}}{\sqrt{\frac{1}{a^2}\left\{\frac{1}{(1-e^2)^2} - \frac{1}{1-e^2}\right\} - \left\{\frac{1}{r} - \frac{1}{a(1-e^2)}\right\}^2}}$$

$$= \frac{d\left\{\frac{a(1-e^2)}{re}\right\}}{\sqrt{1 - \left\{\frac{\frac{a}{r}(1-e^2) - 1}{e}\right\}^2}}$$

If we put $\frac{\frac{a}{r}(1-e^2)-1}{e} = x$ we have

$$dv = -\frac{dx}{\sqrt{1-x^2}}$$

and $$v = \int \frac{-dx}{\sqrt{1-x^2}} = \omega + \cos^{-1} x$$

$$= \omega + \cos^{-1}\left\{\frac{\frac{a}{r}(1-e^2)-1}{e}\right\}$$

ω being an arbitrary constant.

Whence we have

$$\frac{a}{r}(1-e^2) - 1 = e\cos(v-\omega)$$

or $$r = \frac{a(1-e^2)}{1 + e\cos(v-\omega)} \qquad (15)$$

the polar equation of the ellipse.

The angles v and ω are the angles which the radius vector and line of apsides, make with a fixed axis situated in the plane of the orbit or the true longitude of the planet and the longitude of the perihelion respectively. The difference $v - \omega$ determines the angular distance of the planet from perihelion and is called the true anomaly.

If the angle $v - \omega$ is reckoned from the perihelion then $\omega = 0$ and the above equation becomes

$$r = \frac{a\,(1 - e^2)}{1 + e \cos v}. \tag{16}$$

If instead of eliminating dt between equations (9) and (13), we eliminate dv, we shall have

$$\frac{dr^2}{dt^2} + \frac{H^2}{r^2} = \frac{2\mu}{r} + C$$

whence $$dt = \frac{rdr}{\sqrt{C + \frac{2\mu}{r} - \frac{H^2}{r^2}}}$$ and substituting

the values of C and H, we find

$$dt = \sqrt{\frac{a}{\mu}} \cdot \frac{rdr}{\sqrt{2ar - r^2 - a^2\,(1 - e^2)}}$$

$$= \sqrt{\frac{a}{\mu}} \cdot \frac{rdr}{\sqrt{a^2e^2 - (a - r)^2}}$$

which is easily integrated by introducing an auxiliary variable u connected with r by the relation

$$r = a\,(1 - e \cos u) \tag{17}$$

whence $$dr = ae \sin u du$$

and $$dt = \sqrt{\frac{a^3}{\mu}}\,(1 - e \cos u)\,du$$

an expression whose integral is

$$nt + c = u - e \sin u \tag{18}$$

where $$n = \sqrt{\frac{\mu}{a^3}}$$ and c is a constant.

The angle u is called *eccentric* anomaly, nt the *mean* anomaly of the planet (m) and n is the mean motion. When we take for the origin of time the instant the planet is in perihelion, $c = 0$ and we have

$$nt = u - e \sin u \tag{19}$$

If however we reckon the time to commence at any other instant after passing the perihelion nt must be increased by the constant $\varepsilon - \omega$, where ε denotes the mean longitude of the planet at the origin of the time or the longitude at the epoch, and then we shall have

$$nt + \varepsilon - \omega = u - e \sin u \tag{20}$$

Let T be the duration of the sidereal revolution of the planet (m) and a its mean distance from the Sun, then in putting $u = 2\pi$ in (19) we have $nT = 2\pi$

whence
$$T = \frac{2\pi}{n} = 2\pi \sqrt{\frac{a^3}{\mu}} \tag{21}$$

and also
$$\mu = 4\pi^2 . \frac{a^3}{T^2}$$

Eliminating r between (15) and (17) we have

$$\cos u = \frac{e \cos (v - \omega)}{1 + e \cos (v - \omega)}$$

whence
$$\frac{1 - \cos (v - \omega)}{1 + \cos (v - \omega)} = \frac{1 + e}{1 - e} . \frac{1 - \cos u}{1 + \cos u}$$

and
$$\tan \frac{1}{2} (v - \omega) = \sqrt{\frac{1 + e}{1 - e}} . \tan \frac{1}{2} u \tag{22}$$

If the angle v to be reckoned from the perihelion we have $\omega = 0$ and the above becomes

$$\tan \frac{1}{2} v = \sqrt{\frac{1 + e}{1 - e}} . \tan \frac{1}{2} u \tag{23}$$

a formula which enables us to determine the true anomaly in terms of u and also of nt the mean anomaly.

To determine the nature of the conic section described by the body (m) in its motion round the sun, let V = the velocity in the orbit, then by (12) and (13) we have

$$V^2 = \frac{2\mu}{r} + C$$

$$= 2 \left(\frac{\mu}{r} - \frac{1}{a} \right) \text{ since } C = -\frac{\mu}{a} \tag{24}$$

we also have
$$\frac{1}{a} = \frac{2}{r} - \frac{V^2}{\mu}$$

and as a is positive in the ellipse, infinite in the parabola and negative in the hyperbola we conclude that the orbit is an ellipse, a parabola or a hyperbola according as

$$V^2 < \frac{2\mu}{r}, = \frac{2\mu}{r} \text{ or } > \frac{2\mu}{r}$$

We have next to determine the value of k which is constant for the solar system.

From (21) we have $T^2 = 4\pi^2 \frac{a^3}{\mu}$

$$= 4\pi^2 \frac{a^3}{k^2 (1+m)}$$

whence $$k^2 = \frac{2\pi}{T} \cdot \frac{a^{\frac{3}{2}}}{\sqrt{1+m}} \qquad (25)$$

In the case of the Earth, $a = 1$, $T = 365.25638435$ days and the combined mass of the Earth and Moon is $\frac{1}{354710} = m$.

Therefore we easily find

$$\log k = 8.2355814$$

and in seconds of arc

$$\log k'' = 3.5500066$$

or $$k = 3548''.19 = 59' \ 8''.19$$

The quantity $\frac{2\pi}{T}$ is the mean angular motion in a mean solar day and since $\sqrt{1+m}$ differ very little from unity, k is very nearly equal to the mean daily motion of the Earth—a result which might have been anticipated, because the mass of the Sun is just such as will communicate to the Earth the mean daily motion which it actually has.

There is another method of arriving at the equation of the orbit described by the planet, which we will now give for the purpose of making the subject as clear and explicit as practicable. We have already shown (7) that the plane of the orbit passes through the centre of the Sun and if we take this as the plane of reference or the fundamental plane, then $z = 0$, and the general equations (5) of undisturbed motion are

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = 0$$

$$\frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = 0 \qquad (26)$$

and (6) reduces to

$$x\frac{dy}{dt} - y\frac{dx}{dt} = h \tag{27}$$

which transformed into polar coördinates becomes

$$r^2\frac{dv}{dt} = h \tag{28}$$

where h is twice the area described in the time dt.

In polar coördinates we have

$$x = r\cos v \text{ and } y = \sin v$$

then we have

$$\frac{d}{dt}\left(\frac{x}{r}\right) = -\sin v\frac{dv}{dt}$$

$$= -\frac{r\sin v}{r^3}\cdot r^2\frac{dv}{dt}$$

$$= -\frac{y}{r^3}h$$

whence $$\frac{y}{r^3} = -\frac{1}{h}\cdot\frac{d}{dt}\left(\frac{x}{r}\right)$$

Similarly $$\frac{x}{r^3} = \frac{1}{h}\cdot\frac{d}{dt}\left(\frac{y}{r}\right) \tag{29}$$

which substituted in (26) give

$$\frac{d^2x}{dt^2} = -\frac{\mu}{h}\cdot\frac{d}{dt}\left(\frac{y}{r}\right)$$

$$\frac{d^2y}{dt^2} = \frac{\mu}{h}\cdot\frac{d}{dt}\left(\frac{x}{r}\right) \tag{30}$$

By the integration of these we obtain

$$\frac{dx}{dt} = -\frac{\mu}{h}\left(\frac{y}{r} + \beta\right)$$

$$\frac{dy}{dt} = \frac{\mu}{h}\left(\frac{x}{r} + \alpha\right)$$

where a and β are constants.

Substituting in (27) we have

$$\frac{\mu x}{h}\left(\frac{x}{r} + \alpha\right) + \frac{\mu y}{h}\left(\frac{y}{r} + \beta\right) = h$$

or $$\alpha x + \beta y + r = \frac{h^2}{\mu} = \gamma. \text{ (suppose)}] \tag{31}$$

and eliminating r by the relation $r^2 = x^2 + y^2$ and reducing we get

$$(1 - \alpha^2)\, x^2 - 2\alpha\, \beta\, xy + (1 - \beta^2)\, y^2 + 2\alpha\gamma x + 2\beta\gamma y - \gamma^2 = 0 \qquad (32)$$

which is the general equation of the conic sections, the origin of coördinates being evidently at the focus.

The significance of the constants α, β and γ are now to be determined, If we denote by ω the angle which the axis of the conic makes with the axis of x, and by p the distance from the focus to the directrix, the general equation of the conic is

$$(1 - e^2 \cos^2 \omega)\, x^2 - 2e^2 \sin \omega \cos \omega .\, xy + (1 - e^2 \sin^2 \omega)\, y^2 + 2e^2 p \cos \omega .\, x + 2e^2 p \sin \omega .\, y - e^2 p^2 = 0, \qquad (33)$$

and comparing the coefficients of these two equations we easily find

$$e = \sqrt{\alpha^2 + \beta^2}$$

$$\tan \omega = \frac{\beta}{\alpha} \qquad (34)$$

If $\omega = 0$, the axis of x coincides with the axis of the curve, $\beta = 0$ and then (32) becomes

$$(1 - \alpha^2)\, x + y^2 + 2\alpha\gamma x - \gamma^2 = 0$$

and if $x = 0$, $\quad y = \gamma$

$=$ the semi-latus rectum

therefore $\qquad \gamma = \dfrac{h^2}{\mu} \qquad (35)$

When $\beta = 0$, $\alpha = e$ and (31) becomes

$$ex + r = \frac{h^2}{\mu}$$

or in polor coördinates

$$r = \frac{\dfrac{h^2}{\mu}}{1 + e \cos v} \qquad (36)$$

the well known polar equation of the conic sections.

In the preceding formulae (16), (17) and (23) both r and v are connected with nt,—the mean anomaly which we shall henceforth denote by M — through the transcendental equation (19), and in the integration of the equations of disturbed motion it will be necessary to express r and v as functions of M.

Writing M for nt we have from (19)

$$u = M + e \sin u$$

from which we develop cos u, cos $2u$, etc., by Lagrange's theorem, thus

$$F(u) = F(M) + e\left\{f(M)\frac{dF(M)}{dM}\right\} + \frac{e^2}{1.2}\frac{d}{dM}$$
$$\left\{(f(M))^2\frac{dF(M)}{dM}\right\} + \frac{e^3}{1.2.3}\frac{d^2}{dM^2}\left\{(f(M))^3\frac{dF(M)}{dM}\right\} + \text{etc.}$$

Put $F(u) = \cos u$, $F(M) = \cos M$, $f(M) = \sin M$, then we shall have

$$f(M)\frac{dF(M)}{dM} = -\sin^2 M = \frac{\cos 2M - 1}{2}$$

$$\frac{d}{dM}\left\{(f(M))^2\frac{dF(M)}{dM}\right\} = \frac{d}{dM}\left\{-\sin^3 M\right\} =$$
$$-3\cos M + 3\cos^3 M$$
$$= \frac{3}{4}\cos 3M - \frac{3}{4}\cos M$$

$$\frac{d^2}{dM^2}\left\{(f(M))^3\frac{DF(M)}{dM}\right\} = \frac{d}{dM}\left\{-\sin^4 M\right\}$$
$$= -2\cos 2M + 2\cos 4M, \text{ etc.}$$

Substituting in the above we find

$$\cos u = \cos M + \frac{e}{1}\cdot\frac{\cos 2M - 1}{2} + \frac{e^2}{1.2}\left(\frac{3}{4}\cos 3M - \frac{3}{4}\cos M\right)$$
$$+ \frac{e^3}{1.2.3}(-2\cos 2M + \cos 4M) + \text{etc.}$$
$$= \cos M + \frac{e}{2}\cos 2M - \frac{e}{2} + \frac{3e^2}{2^3}\cos 3M - \frac{3e^2}{2^3}\cos M$$
$$+ \frac{e^3}{3}\cos 4M - \frac{e^3}{3}\cos 2M$$
$$= -\frac{e}{2} + \left(1 - \frac{3e^2}{2^3} + \ldots\ldots\right)\cos M$$
$$+ \left(\frac{e}{2} - \frac{e^3}{3} + \ldots\ldots\right)\cos 2M \qquad (37)$$
$$+ \left(\frac{3e^2}{2^3} - \ldots\ldots\ldots\right)\cos 3M$$
$$+ \left(\frac{e^3}{3} - \ldots\ldots\ldots\right)\cos 4M$$
$$+ \quad \ldots\ldots\ldots\ldots\ldots$$

In a similar manner we find

$$\begin{aligned}\cos 2u = &\left(-e + \frac{e^3}{2^2.3} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos M \\ &+ \left(1 - e_5 + \frac{5e^4}{2_5 3} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos 2\,M \\ &+ \left(e - \frac{3^2 e^3}{2^3} + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\text{ccs } 3\,M \quad (38) \\ &+ \left(e^2 - \frac{2^2 e^4}{3} + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos 4\,M \\ &+ (\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot)\end{aligned}$$

and so on.

From (17) and (37) we easily find

$$\begin{aligned}\frac{r}{a} = &\left(1 + \frac{e^2}{2}\right) \\ &+ \left(-e + \frac{3}{2^3}e^3 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos M \\ &+ \left(-\frac{e^2}{2} + \frac{e^4}{3} - \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos 2\,M \quad (39) \\ &+ \left(-\frac{3e^3}{2^3} + \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \right)\cos 3\,M \\ &+ (\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot)\end{aligned}$$

which expresses r in terms of M.

From (28) we have $\dfrac{dv}{dt} = \dfrac{h}{r^2}$

where h is twice the area described in a unit of time, therefore

$$\begin{aligned}Th &= 2 \times \text{area of the ellipse} \\ &= 2\pi a^2 \sqrt{1 - e^2} \\ &= \frac{dM}{dt} \cdot \frac{a^2}{r^2} \sqrt{1 - e^2)}\end{aligned}$$

and $$h = \frac{2\pi}{T} \cdot a^2 \sqrt{1 - e^2}$$

and $$\begin{aligned}\frac{dv}{dt} &= \frac{2\pi}{T} \cdot \frac{a^2}{r^2} \cdot \sqrt{1 - e^2} \\ &= \frac{dM}{dt} \cdot \frac{a^2}{r^2} \sqrt{1 - e^2}\end{aligned}$$

or $$\begin{aligned}\frac{dv}{dM} &= \frac{\sqrt{1 - e^2}}{(1 - e\cos u)^2} \quad \text{by (17)} \\ &= (1 - e^2)^{\frac{1}{2}}(1 - e\cos u)^{-2}\end{aligned}$$

Developing the second member by the binomial theorem and changing the powers of cos u to the cosines of multiples of u we have

$$\frac{dv}{dM} = (1-e^2)^{\frac{1}{2}}\,(1-e\cos u)^{-2}$$

$$= (1 + e^2 + e^4 + \ldots\ldots\ldots\ldots)$$
$$+ 2\,(e + e^3 + e^5 + \ldots\ldots\ldots\ldots)\cos u$$
$$+ \left(\frac{3}{2}e^2 + \frac{7}{2^2}e^4 + \ldots\ldots\ldots\ldots\right)\cos 2u$$
$$+ \left(e^3 + \frac{11}{2^3}e^5 + \ldots\ldots\ldots\ldots\right)\cos 3u$$
$$+ (\ldots\ldots\ldots\ldots\ldots\ldots)\ldots$$

Substituting the values of cos u, from (37) (38), etc., multiplying by dM and integrating we obtain

$$v = M + \left(2e - \frac{e^3}{2^2} + \frac{5e^5}{3.2^5} + \ldots\ldots\ldots\right)\sin M$$
$$+ \left(\frac{5e^2}{2^2} - \frac{11e^4}{3.2^3} + \ldots\ldots\ldots\ldots\right)\sin 2M$$
$$+ \left(\frac{13e^3}{3.2^2} - \frac{43e^5}{2^6} + \ldots\ldots\ldots\ldots\right)\sin 3M$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

No constant is added because $v = 0$ when $M = 0$.

The quantity $v - M$ is the equation of the centre.

For a more elaborate development of these formulæ extended to the 12th power of e and to the 12th multiple of u and M, see my paper in Monthly Notices of the Royal Astronomical Society, Vol. 43, No. 7.

The formulæ just derived enable us to arrive at an approximate integration of the formulæ (4) for disturbed motion.

REPRESENTATIVE STELLAR SPECTRA BY SIR WILLIAM HUGGINS AND LADY HUGGINS.

W. W. PAYNE.

As announced before in this magazine, we have received Vol. 1 of the publications of Sir William Huggins' Observatory at Tulsehill, London, England. The full title of this noble volume is "An Atlas of Representative Stellar Spectra from λ 4870 to λ 3300. There is added a discussion of the evolutional order of the stars,

and an interpretation of their spectra, preceded by a short history of the Observatory and its work. The book is a large quarto in form, printed on fine, heavy plate paper, with some titles and initial letters in red ink. It contains 165 pages with 13 full page plates of representative and most interesting stellar spectra. The plates are beautiful and perfect half-tones in almost every particular and the large scale in which they appear makes them most valuable for comparison or for general reference. The book as a whole is a superb specimen of printing, and is surely very creditably to the publishers, Messrs. Wm. Wesley & Son of London, one of the oldest and best known publishing houses in the world.

On account of its real scientific value, this book ought to find ready sale in America; for certainly scientists, Observatories, scientific libraries and the best general libraries will want a late, authoritative work on stellar spectroscopy by an author who is recognized everywhere as one of the first, if not the leading one in his chosen field of work.

Dr. Huggin's Observatory at Tulse-hill is widely known among astronomers as the pioneer institution in applying the spectroscope to astronomy. It was early in the sixties that Dr. Huggins began this work, and the rapid discoveries which he made have served as the foundation of the new astronomy which has since grown so rapidly and which still promises more and more by the aid of celestial photography. The broad outlook in science thus opened appears to be limitless in the way of better observation, permanent record and more refined and satisfactory measurement. These are the great factors of real progress to which the practical astronomer must ever look with increasing dependence as the limits of his field of labor and discovery are widened and removed farther and farther from the well beaten paths of knowledge.

When Dr. Huggins began to apply the spectroscope to the study of the stars, the difficulties which he met and was obliged to overcome were enormous. These are briefly set out in the first chapter of this volume which, in this regard, is especially good reading for young people who are anxious to do something in new, or comparatively new lines of work, and imagine the task will be an easy one when they once get at it. The experiences of such pioneers who have worked most industriously for nearly two scores of years must contain lessons of great value for those who would walk in like illustrious paths.

It was the announcement of Kirchoff in 1858, that he had discovered the true nature and the chemical conditions of the Sun,

from his interpretation of the Fraunhofer lines that filled the mind of Dr. Huggins with thoughts of a new method for working out some of the hard problems pertaining to the heavenly bodies, which no one, thus far, had been able to solve by the methods of the old astronomy. With the fresh zeal of inventive thought, Dr. Huggins at once wanted to apply the Kirchoff method of the solar spectrum to the study of the stars. He consulted scholarly friends who would be most likely to give him helpful counsel in regard to the probability of success in such an undertaking; but they were uncertain, if not quite sure, of failure, in such an attempt, because they thought the light of the stars too faint for spectra, and because of the Earth's motion which no clock-work was deemed true enough to control for the length of time needed for the delicacy of spectroscopic work. If the Heideberg professor needed all the light of the Sun for the study of the solar spectrum, how could any one hope for success in trying to get the spectrum of the brightest of the stars, when it is remembered that the light of the Sun is 40,000,000,000 times as great as that of Vega, which is more than a first magnitude star. It was at once evident, also, that the prism by which the spectrum was to be secured, could not be placed in front of the object glass of the telescope, as Kirchoff had done, even if the spectrum of any star would be bright enough for observational uses, for it would not then be possible to get at the same time, a spectrum of some terrestrial substance which would serve as a comparison for the sake of identifying that which might appear in a star spectrum. Hence it was necessary to attach the prism to the eye-end of the telescope, and adapt it in such a way that these two spectra needed could be satisfactorily obtained and be definitely compared. This was not all. The telescopic image of a star, if we do not consider the spurious part of it, is only a point, and its spectrum would be ordinarily a narrow line of light without width enough to show the presence of dark lines across it, in a way suitable for study or measurement. To get this necessary breadth of spectrum band was a difficult task. It was finally accomplished by means of a special lens placed in front of the slit of the spectroscope and within the focus of the telescope, that would give just breadth enough to the point-like image of the star needed and no more, for the brightness of the band must diminish as its width increases, and the amount of light for use was too small to expend in that way except as it was absolutely necessary.

So much has been said to give some idea of the problem that our author had to solve in order to make the first stellar spectroscope. It is now so easy a task to obtain fine and complete instru-

ments of this kind that probably astronomers themselves do not often realize the hard work that was done in those early days of pioneering in spectroscopy. It has been well said by another that such work demands a sacrifice of time that is very great when compared with the amount of information that the investigator will be able to obtain; and this is true in some things relating to the early progress of the new astronomy, but it is also true that the first ten years of the new science revealed more new and astonishing things in the way of important discoveries than have come to astronomy in any other like period of time since the invention of the visual telescope.

If we had now the time and space at command, it would well repay us, to set out, in detail, the wonderful history of those early years so admirably penned by Dr. Huggins in the volume before us. But we must be content only to give a passing notice of the more prominent parts of it which are already familiar to those acquainted with the elementary knowledge of spectroscopy.

In 1863 Dr. Huggins gave diagrams of the spectra of Sirius, Betelgeux and Aldebaran in a preliminary note to the Royal Society under the head, "On the Lines of Some of the Fixed Stars," with the statement that he had observed the spectra of some forty stars and also the spectra of Jupiter and Mars. It is an interesting fact to be noted in this connection, that our Mr. Rutherford of New York, independently began the same kind of work, about the same time, and a little later, Sicchi of Rome and Vogel in Germany.

In February of the same year attempts were made to photograph the star spectra, with some success, by the wet colodion process, but the method was so inconvenient that it was not continued, and the photography of stellar spectra was not resumed until 1875. The dry colodion plates were not used at this time because they were not rapid enough for the work desired.

The next thing needed in the new work was a convenient map of the spectra of the chemical elements. Such a map did not then exist. Kirchoff's maps contained only a few elements on an arbitrary scale, relatively to the solar spectrum, and as sunlight could not be used at night for comparison, such a scale was almost useless. To supply this want Dr. Huggins spent the greater part of the year 1863 in mapping the spectra of twenty-six chemical elements with a train of six prisims, the scale used being the spark spetrum of common air, at first carefully referred to that of purified oxygen and nitrogen. With such a map for reference, the work of the following year

went on successfully and rapidly, as one after another the terrestial elements of iron, hydrogen, sodium, magnesium and calcium were easily and certainly found in the substance of the stars, revealing the stupendous fact, that these tiny twinkles of night are great Suns claiming kinship with our own orb of day in the real unity of one vast universe, so beautiful, and so sublime in the Creator's designing thought, that a finite mind stands in awe at the appalling concept.

On August 29, 1864, Dr. Huggins, for the first time, directed his telescope to the planetary nebula in Draco. He was curiously expectant to know if he could gain any knowledge about the nature of the mysterious nebulae which Herschel, Rosse and many others had studied long and profoundly, but without satisfactory results. His description of the first view of the spectrum of a nebulous body is graphic indeed, and is interesting reading, even now after a lapse of thirty six years.

When he looked into the spectroscope, no such view was seen as was anticipated. Only a single bright line at first. Looking a little more closely he saw two other bright lines on the side towards the blue with dark spaces between them. Such a sight from such a source probably never before had been seen by mortal eyes. Can we wonder that Dr. Huggins at first thought that something must be wrong with the spectroscope? But he knew his instrument too well to wait long for the wonderful meaning of this very wonderful discovery. The light from the nebula is truly monochromatic, therefore it is not an aggregation of stars, it is truly a mass of luminous gas.

The great question that had so long baffled the skill of the best scholars was at last solved, but like other important discoveries in science, its answer brought up a hundred others in its place, of less moment to be sure, but all pressing for answer like the first. What is the full meaning of these bright lines? Are the nebulae very hot, or only comparatively so? Are they very far away or only of the same order of distance as the stars. More observation, more thought and the opportunity for more work are the coveted things that await the enthusiastic zeal of those who will press their way into the new and vast fields of research now already so auspiciously opened.

At another time we must say more about Dr. Huggins' early work on comets, variable stars and double stars as well as call attention to the splendid services which the authors of this book have rendered to science in the later years of their lives.

The net price of this new volume is $5. Send to Messrs. Wm. Wesley & Son, 28 Essex Street, Strand, London, England.

MEASUREMENT OF PHOTOGRAPHIC INTENSITIES.

EDWARD C. PICKERING.

The comparison of various celestial and terrestrial sources of light has for many years been a subject for investigation by the officers of this Observatory. One of the first comparisons of the light of the Sun and Moon was made by Professor George P. Bond in 1861 (Mem. Amer. Acad. VIII, N. S , p. 298). Comparisons of the light of the solar corona, of the Moon, and of twilight, with a standard candle were made by the writer in 1870. A comparison of various artificial lights was made by Professor W. H. Pickering, as his graduating thesis in 1879 (Proc. Amer. Acad. XV, 236). Since 1887, all the photographs obtained with the principal instruments of this Observatory have had an image of a standard light photographed on them for purposes of comparison. In 1887, when Mr. W. H. Pickering was placed in charge of the Boyden Department, an important part of his work was a quantitative determination of photographic intensities. This investigation was completed in 1891, and the results are published in the Annals, Volume XXXII, Chapter I. The relative brightness of 10,498 stars, for the wave length 430, was determined by Mrs. Fleming and was published in 1890 in the Draper Catalogue (Annals, Volumes XXVI and XVII). An independent determination of the total photographic brightness of the stars was undertaken in 1896 by photographing stars out of focus. As none of these investigations served to determine certain constants, which seemed to the writer to be important, a more extensive investigation was assigned to Mr. Edward S. King, under whose direction the photographs are taken at Cambridge. This work has been modified and extended by Mr. King, so that it now includes the following researches, which he describes below:—

In 1896, a monthly test of plates was inaugurated, which was of routine character. A complete determination, not only of the sensitiveness of the emulsion, but of the constancy of the light and of the developer employed, is obtained by the expenditure of one 8x10 plate per month (photogram, VI, 261). The means of the results of these tests, extending over a long period of time and necessarily subject to all the accidental errors due to season or other factors, will probably show the effect of the age of the plate on its sensitiveness, whether the image is affected by the interval between the exposure and the development, and other facts of similar interest.

About the same time with the above, the director suggested that an extensive investigation be undertaken of the photographic measurements of various sources of light. The work was to be systematized into a routine, which should occupy the least time in making the observations. By extending the period over years and by performing each experiment under the most diverse circumstances, the final results should be of great accuracy. Preliminary to this work, an apparatus was constructed for making photographic wedges in which equal intervals of the scale corresponded to equal differences in stellar magnitude. The general design was the same as that described in the Annals, Volume XXVI, 6, except that the triangular aperture was replaced by an aperture bounded by a logarithmic curve. The part of the curve extending to minus infinity, but enclosing a finite area, is represented by admitting light through an additional aperture of equivalent area. This apparatus, having a range in aperture of five magnitudes, is useful for measuring the intensity of surfaces directly, or of bright objects, when the aperture is covered with porcelain. It is also of value in comparing the sensitiveness of plates, and for studying the relation of the darkening of the film to the exposure, and to the intensity of the light.

The object of this investigation is to determine the photographic intensity of various sources of light upon a uniform scale. This scale will be that of the Meridian Photometer, in which α Ursæ Minoris has the magnitude of 2.15, and one unit corresponds to the ratio of 2.518, whose logarithm is 0.400. Luminous points may be compared directly with stars. In the case of surfaces, the light emitted by a circle having a diameter of one minute of arc is employed. In general, different portions of the same photographic plate are exposed for a given time to the sources of light to be compared, and the darkening measured with a photographic wedge.

Observations of surfaces will include, intensity of the sky at different distances from the Sun; the sky in the zenith during twilight, on clear and cloudy days, on dark and moonlight nights; comparisons of blue sky with cumulus cloud; intensity of Milky Way, Aurora, and Zodiacal Light. For this purpose pinhole cameras are used with various apertures. For very faint lights, apertures subtending an angle of 60° or more are used. When possible, automatic devices are employed, both to make the exposure and to shift the plate.

For measuring luminous points, the light concentrated by a lens illuminates the plate placed at various distances from the

focus. In this manner, bright stars, the planets, and the Moon at different phases and latitudes, may be compared with α Ursae Minoris or an artificial light placed at a distance. It is also necessary to determine the effect of any slight fogging before, after, or during the principal exposure. The total and local absorption of the lens will also be measured. The same objects are also compared directly by pinhole cameras. In using a pinhole camera to measure points differing greatly in brightness the duration of the exposure must be varied. This necessitates a study of the darkening of the plate with relation to time. The comparison is made, in all cases, with a light, producing an equal darkening in the same time. Another factor may be introduced by inclining the plate. An angel of 60° is equal to a diminution of one-half, or about three-quarters of a magnitude. The exact amount corresponding to the inclination used, must be determined by experiment.

The determination of the light of the Sun is a difficult problem. The light may be reduced by a distant lens of short focus, and compared with the standard light similarly reduced, but at less distance. Another way is by a combination of lenses like a telescope, which enlarges or reduces objects, as we look through one end or the other. The absorption of the instrument is thus eliminated. Two plates of glass, placed so as to give mutiple reflections, will also afford a wide range of exposure, as well as a means of comparing lights of great difference in intensity. Pinhole cameras having the aperture covered with a porcelain plate can be constructed so as to give a range of 15 to 20 magnitudes, which is sufficient for comparing the Sun and the Moon. A second pinhole camera placed in front will cut off the light of the sky from the aperture. The absorption and the diffusing power of the porcelain plate must be measured independently. In all the preceding plans the sensitiveness of the plates to different colors affects the results. By combining a slit spectroscope with the cameras just described, a comparison of the light of the Sun and the Moon may be made in different parts of the spectrum, corresponding to given wave lengths.

The comparison of the spectum of the various sources of light possesses many advantages. The work is placed on such a basis that the results are freed from the troublesome questions of absorption, sensitiveness of the plate, etc., and are rendered directly comparable with those that may be obtained at different times, by other observers, under widely differing circumstances. In fact, the results should be the same as those obtained by the

eye, the bolometer, or in any other manner that may be devised. All these measures may be made either with a telescope and objective prism, or with a slit spectroscope combined with a pinhole camera or telescope. With the objective prism the spectra are made of the same width, either by interposing a cylindrical lens, by moving the plate at the same equable rate for each exposure, or by throwing the image out of focus. In the latter case, a rectangular aperture should be placed over the prism, so as to give width with a minimum loss of definition. With the moving plate, a variation of light is obtained by covering one-half of the prisim at a time, along a line perpendicular to its edge.

The standard light used is an ordinary Argand gas burner shining through a small hole. The star α Ursae Minoris is made the ultimate standard to which all work is referred, since the standard light is compared with it before and after each monthly test of the plates. In addition to the wedges for measuring the density of photographic plates a polarizing photometer is used for comparing surfaces with each other.

Early in 1900, while this work was in progress, Mr. W. H. Pickering, in preparing to observe the Solar Eclipse of May 28, desired to select a suitable plate and developer for the work, and accordingly undertook the following investigation, which he describes below:—

A suitable standard of light has long been wanted in photography. Artificial sources usually give even more uncertain results photographically than they do visually, because a slight variation in temperature will effect the blue end of the spectrum even more than the red end. In 1893 it occurred to the writer to employ as a primary standard of actinic intensity the radiation of a star shining directly upon the photographic plate, without previously passing through or being reflected from any medium, except our atmosphere. Since this standard is too faint for general use, a secondary one to be standardized from it has been devised. The light from the star is condensed through a simple plano-convex lens of 8.2 cm. aperture and short focus, and is focused on a small circle of ground glass 0.5 cm. in diameter. This is placed 3 cm. in front of the sensitive plate, which is exposed to it through a small square aperture measuring 0.2 cm. on a side, cut in a blackened brass plate. The constant of this instrument was determined, and it was found to give about thirty times the light of the direct radiation of a star upon the photographic plate. With twenty minutes exposure α Ursae Minoris darkened the plate sufficiently to produce the "Sensitive

Tint," that is to say the tint where a small variation in the light is most noticeable. The secondary standard, however, is not sufficiently brilliant for ordinary purposes, and a tertiary one has therefore been devised. This consists of a box 50 cm. in length, one end of which contains an aperture 5 cm. in diameter, covered by a piece of ground glass, and the other end carries the sensitive plate and the blackened brass plate described above. Just inside the ground glass may be placed diaphragms ranging from 0.04 to 4.0 cm. in diameter. At a distance of about 200 cm. beyond the ground glass is placed an Argand gas burner. This apparatus gives ample light for all photographic purposes, and the plates exposed in it may always be standardized when necessary by an additional exposure to the secondary standard.

The three pieces of apparatus above described are adapted to various investigations. Thus the photographic brightness of the Moon, and of the brighter stars and planets, may be measured with accuracy by employing different apertures in front of the lens of the secondary standard, and measuring the brightness of the various images obtained by means of a photographic wedge, or a series of standard squares of varying density. It is very important to vary the aperture rather than the time of exposure, since the results obtained in the latter case would have to be corrected by the "time Correction" (Annals, Volume XXXII, 20). The secondary standard also enables us to express the sensitiveness of a plate, in terms of universal application throughout all time. Thus the Seed plate No. 27 is capable of being appreciably darkened when exposed for ten minutes to a source equal to 300 times the brightness of α Ursae Minoris. This amount of light is equal to about thirty times the brightness of a star whose photographic magnitude is 0.0. The logarithm of this number, 1.5, may be conveniently used to represent the sensitiveness of the Seed plate. By the use of the secondary and tertiary standard, together with an ordinary photographic telescope, we may make a study of the brightness of nebulae, comets and other luminous surfaces.

From these various investigations it is hoped that we shall obtain a scale of photographic intensities with which all sources of light may be compared and to which they may be referred.

HARVARD COLLEGE OBSERVATORY Circular No. 50.

May 9, 1900.

NON-EUCLIDEAN GEOMETRY.

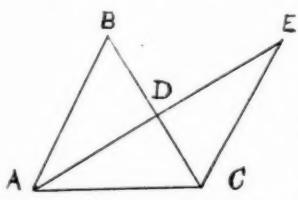
W. H. S. MONCK.

FOR POPULAR ASTRONOMY.

The question of the universal validity of Euclid's axiom regarding parallel lines was one which exercised a good deal of my attention many years ago when I was a student. I came to the conclusion that strict proof was impossible, but that the axiom itself was true and that the objections to it arose either from not realizing its real import in the imagination, or from losing sight of the fact that we were dealing with the real properties of space and that, as always happens when we apply algebraic or symbolical reasonings to real objects, we might readily carry this symbolism too far. At this time I took no interest in astronomy. Indeed the book then used in Dublin University would hardly have led any one to think that astronomy was a physical science of a very progressive character.

But one thing it seemed to me that Legendre did establish without the aid of Euclid's famous axiom viz: that the three angles of a plane triangle could not *exceed* two right angles. The difficulty was to prove that they could not be *less*—perhaps even less than one right angle. The proof that they cannot be greater is substantially the following:

Let ABC be any triangle and let C be its greatest angle and B the next in magnitude. Draw AD bisecting BC and make DE equal to AD and join CE. Now the triangle EDC is evidently equal in all respects to the triangle ADB, and the sum of the three angles of the triangle AEC is evidently equal to that of the original triangle ABC. But the angle ACE is equal to the sum of the two greates tangles of the original triangle ABC viz: ACB and ABC. We can repeat this process as often as we like (bisecting AC instead of CE next time, if that course will increase the angle at C more rapidly), and by carrying this proces on sufficiently far we can obtain a triangle the sum of whose three angles is the same as that of the original triangle, but the two smaller angles in which taken together do not amount to one-millionth (or one hundredth millionth) part of the smallest of the three angles of ABC.



It seemed to me also that the failure to prove Euclid's axiom

in the terms laid down arose from the difficulty of applying the principle of superposition to infinites. Let us suppose a number of right lines drawn from a centre and produced to infinity, the angles between each successive pair being equal. Then if the principle of superposition were applied each adjacent pair of lines would enclose an equal space, and if there were a million such lines each pair would enclose one millionth part of the whole plane extended to infinity. Now on the contrary take a pair of right lines crossing a third in such a manner that the two interior angles are together equal to two right angles.

Let AB* be the intersected line and let the two intersecting lines be CF and DG. Take $DE = CD$ and draw EH making the angle DEH = the angle CDG. Suppose all four lines produced to infinity towards the right. Then applying the principal of superposition the space enclosed between the CF and DG and that enclosed between the lines DG and EH (to the right of the intersecting line AB) will be equal. But if AB be produced to infinity we can mark off as many segments equal to CD upon it as we please and draw lines under similar conditions through the concluding point of each segment. Consequently the space included between the pair of lines CF, DG produced to infinity and the intersecting line AB does not bear any finite proportion to the whole plane extended to infinity.

Then draw CI such that the two angles CDG, DCI are together less than two right angles, reduce all the lines to infinity. The space included in the angle FCI is greater than that included in between the lines CF and DG since the former bears a finite ratio to the entire plane extended to infinity while the latter does not. But if the line CI does not intersect the line DG the former space is included in the latter space and we should have a part greater than the whole. Consequently if the principle of superposition is applicable to infinity Euclid's axiom holds good.

But it will be said that our experience can never extend to infinity and we are therefore not warranted in extending any principle of infinity. I do not desire to enter into the metaphysics of the question. I will only say that what holds good within the limits of our experience may be fairly assumed to hold good beyond these limits until we see some reason to the contrary. And

* By mistake this cut was not engraved. The reader can easily supply it by reference to the figure on page 336 with the following suggestions: AB is the same in both figures, if A and B be placed respectively a little higher and lower than at the points of intersection of the lines. In the wanting cut CF is the same as AC, CI as AE, and EH as BD. Then draw a parallel line, DG, midway between CF and EH and the new figure is complete.—Ed.

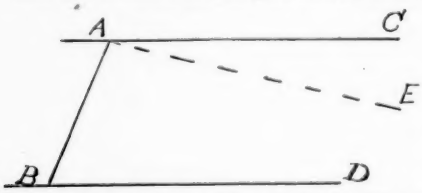
I think if Euclidean geometry did not hold good with respect to the most distant stars that we have tried, its failure ought to be brought to light by our researches on parallax. If the three angles of a triangle with very long sides fell short of 180° we should probably find a considerable parallax for any very remote star; while if it exceeded 180° we ought to discover many more negative parallaxes—perhaps of considerable amount—than we have done. Our researches are as far as I know grounded on the assumption that the three angles of the triangle (whose base consists of the line going two points on the earth's orbit and whose base angles are deduced from observation) are equal to 180°. Nor is this the only instance in which such an assumption is made; but it is not necessary to enter into details.

Although I am a firm believer in Euclidean geometery I am quite prepared to accept any evidence derived from observations on stars presumably very distant which seems to be irreconcilable with it. If Euclid be wrong the stars alone can prove him to be so and I think that our present knowledge of the stars has progressed far enough to throw considerable light on the subject. I should therefore be glad to hear from some non-Euclidean what are the observed facts of astronomy which are relied on as inconsistent with Euclidean geometry—a kind of geometry whose validity as regards terrestrial measurements seems to be established by ample experience.

Let me add however that in my opinion Euclid's geometry is not a mere logical deduction from his assumptions. He appeals at every step to our knowledge of space but this knowledge is so simple and universal that the reader is apt to overlook the fact that Euclid is always dealing with realities and not with mere argument. If you cannot draw the figure or draw it wrongly you cannot follow him. If when you were asked to bisect the base of a triangle by a right line from the vertex, you drew the line outside of the triangle and made the connection below the base, the conclusion will probably not follow. This, it may be said, would not be a right line. But can you prove this from Euclid's definition of a right line? If there were such a being as a man who had no idea of space he would find himself utterly unable to deduce any conclusion from Euclid's definitions and axioms. Euclid no doubt assumes that the properties of space are the same everywhere and at all times; but his constant appeal is, "Draw your figure. Test what I say by the space which you know and experience. You will not see the force of my reasonings until you do that."

I have treated Euclid's axiom as an assumption that the three angles of every plane triangle are equal to two right angles. If that fact could be proved otherwise the axiom might be dispensed with. Euclid puts it to no other use. Various forms of the axiom have been given with the view of making it more acceptable. Its acceptance I think does not depend on the terms but on the the realization of their import by the hearer.

Let the sum of the angles *BAC*, *ABD* be equal to two right angles (180°). It is susceptible of proof that the right lines *AC* and *BD* cannot meet. Draw the right line *AE*. Can you believe that however both lines may be produced *AE* will never meet *BD*? The difficulty of belief does not arise from the terms employed but from gazing at or contemplating the figure.



SPECTROSCOPIC NOTES.

The entire path of totality of the eclipse of May 28 has been favored with a perfect sky. European observers in Portugal, Spain, and Algiers report clear weather. In this country all along the central line weather conditions were magnificent; the good fortune of which is accentuated by the fact of partial cloudiness as near to the path as New York.

The instrumental equipment, which was elaborate, was installed chiefly at stations in North Carolina, Georgia and Alabama. Wadesboro, N. C., Pinehurst, N. C., Thomaston, Ga., Barnesville, Ga., Greenville, Ala., and Fort Deposit, Ala., might perhaps be mentioned as favorite points. Of the congregations of visiting sight-seers that at Norfolk, Va., was undoubtedly the most prominent.

The amount of apparatus was large, and its variety considerable, including instruments adapted to almost every possible line of research, visual, photographic, spectroscopic, bolometric. Of the spectroscopic results a few only have been reported through the medium of the daily press; the great mass being largely spectrographic, must await development of plates and subsequent publication in the scientific journals.

M. Paulsen (*Comptes Rendus*, March 5; *Nature*, April 26) during brilliant auroral displays in Iceland from Dec. 31 to Jan. 25 has succeeded in photographing a number of new bright lines in the violet and ultra-violet of the spectrum of the aurora.

The statement has been in general circulation that spectroscopic evidence of the rotation of Venus has at last been secured. The accuracy of measurement of rotation in the line of sight has for some time been adequate to the detection of

the rotation of Venus if the period is as short as a day; in fact the persistent failure to find evidence of rotation has perhaps come to be regarded as evidence on the side of those who contend for a period of 225 days. If the planet rotates in a day the velocity would be about a third of a mile per second. The effective velocity at time of superior conjunction would be two-thirds of a mile per second, or, as near conjunction as observations could be taken, perhaps half a mile per second; a quantity at present measurable. For the present the announcement of Belopolsky's results goes no further than the bare statement that the period of the planet's rotation is found to be short.

In *Astronomische Nachrichten* No. 3633 Mr. Epsin publishes a further liberal list of stars with remarkable spectra. The stars are with one exception fainter than the seventh magnitude, and most of them are more or less striking examples of type III. A large majority of the stars of the present list are situated in the Milky Way near Cygnus.

At the meeting of the Royal Astronomical Society, March 9, Mr. Shackleton (*Monthly Notices*, March; *Observatory*, April) read a paper describing his apparatus for examining the distribution of matter in the sun's corona. A photograph was exhibited, taken near the middle of totality in the eclipse of 1896, showing the continuous spectrum to a distance of 15′ from the moon's limb, while the coronium line at λ 5303 reached a height of only 5′. Mr. Shackleton described a combination of color screens admitting a narrow band of green light in the neghborhood of λ 5303 and suppressing the rest of the spectrum. If coronium is confined to the inner corona, photographs taken through such a screen chiefly by the coronium radiation should be less extended and less irregular in form than ordinary photographs in which the entire continuous spectrum of the outer corona is active.

Prof. FitzGerald contributes to *Nature* of May 3 a short note in support of the idea that the sun's corona is of the nature of a solar aurora. He cites, to explain the absence of a dark line of coronium in the sun's spectrum, recent experiments by Herr Cantor which seems to show that when the radiation of a gas is due to electric discharge there is no corresponding absorption.

Mr. Newall contributes to the *Monthly Notices of the Royal Astronomical Society* for March some results of his study of the spectroscopic binary Capella. His former period of 104 days is confirmed. The observations show that the two components of different types of spectrum are nearly equal in mass, and are not very different in brightness. The radius of the relative orbit if it is seen edgewise is about 52,000,000 miles, or if seen inclined at an angle of 60° to the line of vision 104,000,000. Assuming as accurate Elkin's rather uncertain value of 0″.08 for the parallax, the actual brightness is about 500 times that of the Sun; for the orbit assumed to be seen edgewise the angular separation is 0″.04 and the mass 1.7 that of the Sun; for the orbit assumed to be seen under an angle of 60° the angular separation is 0″.08 and the mass 14 that of the Sun.

PLANET NOTES FOR JUNE AND JULY.

H. C. WILSON.

Mercury is evening star with Venus and will be visible to the naked eye at least during the first week in July. On June 22 in the morning Mercury will be in conjunction with Venus, Mercury being then 2°19′ north of Venus. Mercury

THE CONSTELLATIONS AT 9 P. M., JULY 1, 1900.

and the new moon will be near each other on the night of June 28, and again on July 26. On the morning of July 4 Mercury will be at greatest elongation east from the Sun, 26° 1′. Toward the end of the month Mercury will approach the Sun, coming to inferior conjunction on the morning of August 1.

Venus will be visible as evening star only a few days more, since she will be at inferior conjunction on the morning of July 8. A few days after that one may look for the planet in the north-east in the morning.

Mars crosses the meridian between nine and half past nine and can only be studied now in the morning twilight. He is so nearly keeping pace with the earth in his orbit around the Sun that the relative positions change slowly.

Jupiter is at his best for this year, visible toward the south in Scorpio in the early evening. He will be stationary in right ascension July 28 and after that move eastward among the stars.

Saturn will be at opposition June 23 in the constellation Sagittarius. You see this planet every evening now, toward the southeast after nine o'clock, a golden star with steady light. It is a pity that, now that the rings are turned as open to us as they ever are, the altitude of the planet should be so low that observations are always unsatisfactory. Our loss, however, is gain to those in the southern hemisphere and they ought to have their turn at this splendid planet.

Uranus is in Scorpio and about 7° east and 2° south of Jupiter and may be seen at the same hours.

Neptune is behind the sun.

Phenomena of Jupiter's Satellites.

Central Standard Time.

	h	m				h	m		
June 22	9	01 P. M.	II	Tr. In.	July 9	10	28 P. M.	I	Sh. In.
	10	14 "	II	Sh. In.		11	46 "	I	Tr. Eg.
	11	27 "	II	Tr. Eg.	10	7	10 "	II	Sh. Eg.
23	12	43 A. M.	II	Sh. Eg.		9	56 "	I	Ec. Re.
24	12	10 "	I	Sh. In.	11	7	11 "	I	Sh. Eg.
	1	44 "	I	Tr. Eg.	13	8	54 "	III	Oc. Dis.
	2	23 "	I	Sh. Eg.		10	51 "	III	Oc. Re.
	8	49 P. M.	I	Oc. Dis.	15	10	24 "	II	Oc. Dis.
	11	39 "	I	Ec. Re.	16	11	22 "	I	Tr. In.
25	8	11 "	I	Tr. Eg.	17	7	14 "	II	Sh. In.
	8	52 "	I	Sh. Eg.		7	35 "	II	Tr. Eg.
26	12	03 A. M.	III	Tr. In.		8	37 "	I	Oc. Dis.
29	11	18 P. M.	II	Tr. In.		9	44 "	II	Sh. Eg.
30	12	48 A. M.	II	Sh. In.		11	51 "	I	Ec. Re.
	1	44 "	II	Tr. Eg.	18	8	02 "	I	Tr. Eg.
July 1	9	41 P. M.	II	Ec. Re.		9	06 "	I	Sh. Eg.
	10	35 "	I	Oc. Dis.	24	7	32 "	II	Tr. In.
2	1	33 A. M.	I	Ec. Re.		8	45 "	III	Sh. Eg.
	7	45 P. M.	I	Tr. In.		9	49 "	II	Sh. In.
	8	33 "	I	Sh. In.		10	00 "	II	Tr. Eg.
	9	58 "	I	Tr. Eg.		10	27 "	I	Oc. Dis.
	10	47 "	I	Sh. Eg.	25	12	19 A. M.	II	Sh. Eg.
3	8	02 "	I	Ec. Re.		7	39 P. M.	I	Tr. In.
6	7	18 "	III	Oc. Re.		8	48 "	I	Sh. In.
	8	57 "	III	Ec. Dis.		9	53 "	I	Tr. Eg.
	10	49 "	III	Ec. Re.		11	01 "	I	Sh. Eg.
8	8	00 "	II	Oc. Dis.	26	8	14 "	I	Ec. Re.
9	12	18 A. M.	II	Ec. Re.	31	7	54 "	III	Tr. Eg.
	12	22 "	I	Oc. Dis.		9	58 "	II	Tr. In.
	9	33 "	I	Tr. In.		10	37 "	III	Sh. In.

NOTE.—In. denotes ingress; Eg., egress; Dis., disappearance; Re., reappearance; Ec., Eclipse; Oc., occultation; Tr., transit of the satellite; Sh., transit of the shadow.

Jupiter's Satellites for July.

Phases of the Eclipses of the Satellites for an Inverting Telescope.

I.

III.

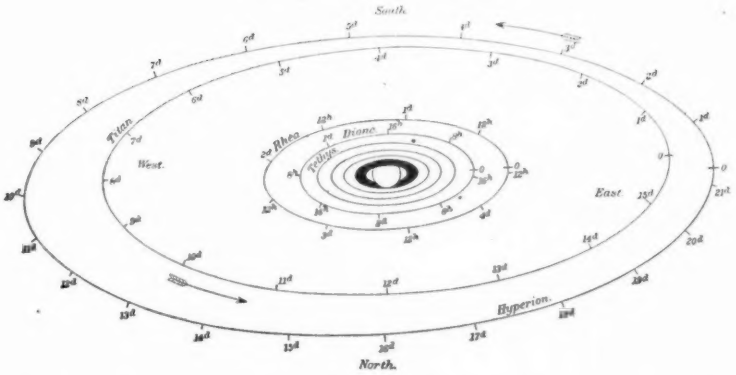
II.

IV. No Eclipse.

Configurations at 9h for an Inverting Telescope.

Day.	West.	East.
1	·4 ·1	○ ·2 3·
2	○ 1· ·4	○ 3· 2·
3	3· 2·	○ ·1 ·4
4	3· ·2 1·	○ ·4
5	·3 ·	○ ·1 ·2 ·4
6	·1	○ ·3 2· ·4
7	2·	○ 1· ·3 4·
8	1·	○ 3· 4· ·2●
9		○ 1· 3· 2· 4·
10	3· 2·	○ ·1 4·
11	3· ·2 1· 4·	○
12	4· ·3	○ ·1 ·2
13	4· 1·	○ 2· ·3●
14	4· 2·	○ 1· ·3
15	·4 ·1 ·2	○ 3·
16	·4	○ 1· 3· 2·
17	·4 3· 2·	○ .1●
18	3· ·4 ·2 1·	○
19	·3	○ ·1 ·2 ·4●
20	1· ·3	○ 2· ·4
21	2·	○ ·1 ·3 ·4
22	·1 ·2	○ 3· ·4
23		○ 1· ·2 ·3 ·4
24	○ 2· 3· ·1	○ 4·
25	○ 1· 3· ·2	○ 4·
26	·3	○ ·1 ·2 4·
27	·3 1·	○ 4· 2·
28	2· 4·	○ ·1 ·3
29	4· ·2 ·1	○ ·3
30	4·	○ 1. ·2 3·
31	4· ·1 3·	○ 2·

The Satellites of Saturn.



APPARENT ORBITS OF THE SEVEN INNER SATELLITES OF SATURN, AT OPPOSITION IN 1900, AS SEEN IN AN INVERTING TELESCOPE.

I. MIMAS.

Period $0^d\ 22.6^h$

	h		
June 23	2.3	A. M.	W
24	12.9	"	W
24	11.6	P. M.	W
25	10.2	"	W
26	8.8	"	W
27	7.4	"	W
July 1	2.5	A. M.	E
2	1.2	"	E
2	11.8	P. M.	E
3	10.4	"	E
4	9.0	"	E
5	7.6	"	E
6	6.3	"	E
10	1.4	A. M.	W
10	12.0	Midn	W
11	10.6	P. M.	W
12	9.2	"	W
13	7.9	"	W
14	6.5	"	W
18	1.6	A. M.	E
19	12.2	"	E
19	10.8	P. M.	E
20	9.5	"	E
21	8.1	"	E
22	6.7	"	E
27	12.5	A. M.	W
27	11.1	P. M,	W
28	9.7	"	W
29	8.3	"	W
30	6.9	"	W

II. ENCELADUS.

Period $1^d\ 08.9^h$

	h		
June 21	5.9	A. M.	E
22	2.7	P. M.	E
23	11.6	"	E
25	8 5	A. M.	E
26	5.3	P. M.	E
28	2.2	A. M.	E
29	11.1	"	E
30	8.0	P. M.	E
July 2	4.9	A. M.	E
3	1.7	P. M.	E
4	10.6	"	E
6	7.5	A. M.	E
7	4.4	P. M.	E
9	1.2	A. M.	E
10	10.1	"	E
11	7.0	P. M.	E
13	3.9	A. M.	E
14	12.7	P. M.	E
15	9.6	"	E
17	6.5	A. M.	E
18	3.4	P. M.	E
20	12.2	A. M.	E
21	9.1	"	E
22	6.0	P. M.	E
24	2.9	A. M.	E
25	11.8	"	E
26	8.7	P. M.	E
28	5.5	A. M.	E
39	2.4	P. M.	E
30	11.3	"	E

III. TETHYS.

Period $1^d\ 21.3^h$

	h		
June 21	2.4	A. M.	E
22	11.7	P. M.	E
24	9.0	"	E
25	6.3	"	E
28	3.6	"	E
30	12.9	"	E
July 2	10.1	A. M.	E
4	7.4	"	E
6	4.7	"	E
8	2.0	"	E
9	11.3	P. M.	E
11	8.6	"	E
13	5.9	"	E
15	3.2	"	E
17	12.5	"	E
19	9.7	A. M.	E
21	7.1	"	E
23	4.3	"	E
25	1.6	"	E
26	10.9	P. M.	E
28	8.2	"	E
30	5.5	"	E

IV. DIONE.

Period $2^d\ 17.7^h$

	h		
June 20	6.9	A. M.	E
23	12.6	"	E
25	6.3	P. M.	E
28	11.9	A. M.	E
31	5.5	"	E

DIONE, Con't.

		h		
July	3	11.2	P. M.	E
	6	4.9	"	E
	9	10.5	A. M.	E
	12	4.1	"	E
	14	9.8	P. M.	E
	17	3.5	"	E
	20	9.1	A. M.	E
	23	2.8	"	E
	25	8.4	P. M.	E
	28	2.1	"	E
	31	7.7	A. M.	E

V. RHEA.

Period 4^d 12.5^h

		h		
June	19	5.5	P. M.	E
	24	5.8	A. M.	E
	28	6.2	P. M.	E
July	3	6.5	A. M.	E
	7	6.8	P. M.	E

RHEA, Con't.

		h		
	12	7.1	A. M.	E
	16	7.4	P. M.	E
	21	7.8	A. M.	E
	25	8.1	P. M.	E
	30	8.5	A. M.	E

VI. TITAN.

Period 15^d 23.3^h

		h		
June	20	8.9	P. M.	W
	24	6.9	"	S
	28	3.9	"	E
July	2	4.2	"	I
	6	6.2	"	W
	10	4.2	"	S
	14	1.4	"	E
	18	2.2	"	I
	22	3.7	"	W
	26	1.7	"	S
	30	11.0	A. M.	E

VII. HYPERION

Period 21^d 7.6^h

		d	
June		19.4	W
		25.2	S
		30.4	E
July		5.2	I
		10.6	W
		16.5	S
		21.7	E
		26.5	I
		31.9	W

VIII. JAPETUS.

Period 79^d 22.1^h

		d	
June		12.7	I
July		2.9	W
		23.0	S
Aug.		10.9	E

VARIABLE STARS.

J. A. PARKHURST.

Maxima and Minima of Long Period Variables.

1900 August and September.

MAXIMA.

	Aug.
—Aurigae	4
V Orionis	4
V Virginis	4
R Tauri	5
SS Cygni	8?
RR Scorpii	8
U Draconis	11
S Orionis	14
R Ophiuchi	15
R Vulpeculae	16
RR Librae	18
RZ Scorpii	20
U Eridani	23
R Pyxidis	23
RT Librae	24
V Tauri	24
R Leonis minoris	25
R Aquilae	26
U Cygni	26
X Geminorum	26
RR Capricorni	28
R Bootis	29
R Corvi	29

MAXIMA, Con't.

	Sept.
S Herculis	1
T Arietis	2
X Puppis	3
T Herculis	4
R Serpentis	4
R Aurigae	5
T Centauri	6
V Canis min.	7
S Lyrae	7
RW Scorpii	7
—Aquilae	8
S Ursae Maj.	8
RU Librae	17
R Lyncis	19
Z Scorpii	19
T Serpentis	21
R Canis min.	22
W Hydrae	22
R Scorpii	22
RU Herculis	26
S Bootis	28
T Draconis	20
S Hydrae	30
W Scorpii	30

MINIMA.

	Aug.
W Persei	4
U Aurigae	5
U Piscium	7
T Ursae Majoris	8
RR Sagittarii	10
R Leporis	11
T Aquarii	14
Y Capricorni	17
R Microscopii	18
X Ophiuchi	19
Z Cygni	20
R Arietis	21
T Andromedae	22
R Cancri	28
	Sept.
R Trianguli	1
S Delphini	5
S Camelopard.	7
W Herculis	8
η Geminorum	10
S Leonis	12
R Canum Ven.	17
U Herculis	17
X Librae	20
U Bootis	26

NOTES TO LONG PERIOD EPHEMERIS.—The star "—Aurigae" is Ceraski's variable at $5^h\ 20^m\ 9^s$, $+36°\ 49'$, (1900.) For chart see this magazine, Vol. VII, page 43.

The star "—Aquilae" is Anderson's variable at $20^h\ 8^m\ 3^s$, $+12°\ 41'.7$, (1900.) See this magazine, Vol. VI, page 532 and VII, 265.

The star "W Persei" is V Persei in Chandler's Third Catalogue.

Minima of the Variable Stars of the Algol Type.

(Given to the nearest hour in Greenwich Time.)

1900.

ALGOL.

	d	h
July	3	23
	6	20
	9	17
	12	13
	26	21
	29	18
Aug.	1	15
	4	12
	18	20
	21	17
	24	14
	27	10

λ TAURI.

	d	h
July	4	19
	8	18
	12	17
	16	16
	20	14
	24	13
	28	12

U CORONAE.

	d	h
July	10	15
	17	13
	24	10
	31	8
Aug.	10	16
	17	14
	24	12
	31	10

δ LIBRAE.

	d	h
July	1	8
	3	16
	8	18
	10	16
	15	8
	17	15
	22	7
	24	15
	31	15
Aug.	7	14
	14	14
	21	13
	28	13

Y CYGNI.

$2P = 2^d\ 23^h.9.$

Even min.

	d	h
July	2	14
Aug.	1	13

Odd min.

	d	h
June	30	17
July	30	16
Aug.	29	15

Z HERCULIS.

$2P = 3^d\ 23.8^h.$

Even min.

	d	h
July	1	13
Aug.	2	11

Odd min.

	d	h
July	3	13
Aug.	4	11

U CEPHEI.

	d	h
July	3	9
	5	20
	8	8
	10	20
	13	8
	15	20
	18	8
	20	19
	23	7
	25	19
	28	7
	30	19
Aug.	2	6
	4	18
	9	18
	14	18
	19	17
	24	17
	29	17

W. DELPHINI.

	d	h
July	2	13
	7	9
	21	19
	26	14
	31	9
Aug.	14	19
	19	15
	24	10

+ 45°3062.

	d	h
July	2	4
	6	18
	11	8
	15	21
	20	11
	25	1
	29	15
Aug.	3	4
	7	18
	12	8
	16	22
	21	11
	26	1
	30	15

U OPHIUCHI.

$P = 0^d\ 20^h.1.$

	d	h
June	30	6
July	31	7
Aug.	31	8

R CANIS MAJ.

P $1^d\ 3^h.3.$

	d	h
June	30	8
Aug.	1	3
	31	19

SS CYGNI—After a normal period which had lasted 33 days SS Cygni began rising about 1900 April 25.0, passed 9.35 magnitude Apr. 25.7, passed maximum Apr. 27, and reached normal light again May 6.0. The maximum was as short as any on record, 11 days; the average of 8 short maxima being 12.2 days. As the order of maxima has been reversed, those of January and March 1900, both being long, it is a question of much interest what course the star will now

follow. The past maximum was covered by 12 observations by Mr. Zaccheus Daniel and 3 by the writer.

AN UNUSUALLY FAINT VARIABLE has been discovered by Prof. Schwassmann and announced in No. 3636 of the *Nachrichten*. The following magnitudes were determined from photographs—

1892 April 17 not brighter than 14.
1893 April 14 about 10.2.
1894 March 29 not brighter than 14.
1896 April 16 about 11.2.
1900 April 2 not brighter than 14.

The place of the new variable is—

R.A. $13^h\ 2^m\ 39^s.5$, Dec. $-12°\ 37'\ 50''$, (1900.)

COMET AND ASTEROID NOTES.

Ephemeris of Comet *a* 1900.

1900		α h	m	s	δ °	′	log *r*	log Δ
June	20	0	26	8	+ 36	29.9	0.1874	0.1886
	21		23	0	36	59.5		
	22		19	43	37	29.4	0.1916	0.1785
	23		16	16	37	59.5		
	24		12	38	38	29.7	0.1958	9.1683
	25		8	50	39	0.1		
	26	0	4	50	39	30.6	0.2000	0.1580
	27	23	0	40	40	1.1		
	28		56	17	40	31.5	0.2042	0.1478
	29		51	42	41	1.7		
	30		46	53	41	31.8	0.2085	0.1376
July	1		41	50	42	1.5		
	2		36	32	42	30.7	0.2128	0.1277
	3		30	59	42	59.4		
	4		25	11	43	27.4	0.2172	0.1181
	5		19	7	43	54.6		
	6		12	47	44	20.7	0.2216	0.1090
	7	23	6	10	44	45.6		
	8	22	59	17	45	9.2	0.2260	0.1003
	9		52	8	45	31.2		
	10		44	43	45	51.5	0.2304	0.0924
	11		37	2	46	9.8		
	12		29	5	46	25.9	0.2349	0.0850
	13		20	54	46	39.7		
	14		12	29	46	50.9	0.2393	0.0786
	15	22	3	52	46	59.4		
	16	21	55	5	47	5.1	0.2438	0.0733
	17		46	8	47	7.7		
	18		37	4	47	7.1	0.2483	0.0692
	19		27	55	47	3.2		
	20		18	42	46	55.9	0.2528	0.0662
	21		9	29	46	45.1		
	22	21	0	16	46	30.8	0.2573	0.0645
	23	20	51	6	46	13.0		
	24		42	2	45	51.8	0.2617	0.0644
	25		33	6	45	27.4		
	26		24	19	45	0.0	0.2662	0.0658
	27		15	43	44	29.6		
	28	20	7	20	43	56.4	0.2706	0.0686
	29	19	59	11	43	20.6		
	30		51	15	42	42.4	0.2750	0.0728
	31	19	43	37	+42	2.1		

Ephemeris of Eros.

[By James B. Westhaver from Elements by Henry Norris Russell; from *Astr. Journal* No. 479.]

Greenwich Midn.		R. A. h m s	Decl. ° ′ ″	log Δ	Mag.
May	27	23 46 17.5	+ 2 39 4	0.26387	13.1
	29	49 53.3	3 13 22	0.25883	
	31	53 28.4	3 47 48	0.25370	13.0
June	2	23 57 2.6	4 22 22	0.24848	
	4	0 0 36.1	4 57 6	0.24317	12.9
	6	4 8.7	5 31 57	0.23778	
	8	7 40.7	6 6 57	0.23230	12.9
	10	11 11.9	6 42 6	0.22674	
	12	14 42.5	7 17 24	0.22108	12.8
	14	18 12.4	7 52 51	0.21533	
	16	21 41.7	8 28 27	0.20949	12.7
	18	25 10.3	9 4 12	0.20355	
	20	28 38.1	9 40 5	0.19752	12.7
	22	32 5.3	10 16 8	0.19140	
	24	35 31.8	10 52 19	0.18518	12.6
	26	38 57.5	11 28 39	0.17886	
	28	42 22.4	12 5 8	0.17244	12.5
	30	45 46.6	12 41 47	0.16593	
July	2	0 49 10.0	+ 13 18 34	0.15932	12.5

The asteroid was picked up by Professor Howe, of Denver, on the night of May 27, the position being

May 27.9129 R. A. 23^h 48^m $3^s.9$ Decl. + 2° 46′ 33″.

From this it may be seen that the planet is about 1^m in advance of its predicted place in right ascension and very close in declination.

GENERAL NOTES.

The next number of this publication, which will be for the months of August and September will be mailed about the 20th of August.

We have received so many articles about the total solar eclipse of May 28, 1900, that we could not publish all this time. Considerable more will appear in our next number.

The Spectrum of the "Flash."—We have had strong hope that some of the many parties who observed the total solar eclipse of May 28 last, in America, would obtain photographs of the spectrum of the "flash" so-called. We know this work was planned for by some parties, but up to this writing, we have not heard that any one succeeded. Along the entire path of totality in the United States, the oppertunity for all kinds of observation was exellent. It is not yet known why everyone was unsuccessful. For next number we hope to have the results of all important eclipse work done at home or abroad.

Observations of Eros—A letter received from the Arequipa Station of this Observatory gives details concerning four photographs of Eros taken there in April with the Bruce Telescope by Dr. Delisle Stewart.

An adjacent star was followed in an eyepiece, and by means of a micrometer

screw the photographic plate was moved with regard to it by an amount and in a direction equal to the motion of Eros. The stars thus appeared as trails and Eros as a point. Approximate positions were determined from the plates at Arequipa with the results given below. Paper print of two of these plates were sent to Cambridge and measures of them are also given. The negatives are now on their way to Cambridge and as soon as received, accurate position will be derived from them.

Plate.	Date 1900.	G. M. T. h m	R. A. 1900. h m s	Dec. 1900. ° ′
A 4333	April 26	21 20	22 49 21	−5 46 0
"	"	"	49 23	−5 46.4
A 4334	"	22 06	49 27	−5 45.8
A 4338	April 27	21 47	51 23	−5 29.6
A 4341	" 30	21 16	57 7	−4 42.1
"	" "	"	57 1	−4 42.3

These appear to be the first observations of Eros since its conjunction with the Sun. The second observations taken a month later are given in the accompanying bulletin. Efforts have been made here to observe Eros both visually and photographicaly, but have failed owing to twilight.

HARVARD COLLEGE OBSERVATORY, Cambridge, Mass.,
June 1, 1900.

Observations of the Eclipse of May 28, 1900.—The following observations of contacts were made at the Chamberlin Observatory, University Park, Colo., under favorable atmospheric conditions. The position of the Observatory is given in the American Ephemeris, under the name Denver.

FIRST CONTACT.

Greenwich M. T.			Instrument.	Observer.
0^h	44^m	$22^s.6$	Twenty-inch	Herbert A. Howe.
0	44	25 .5	Six-inch	Chas. J. Ling.

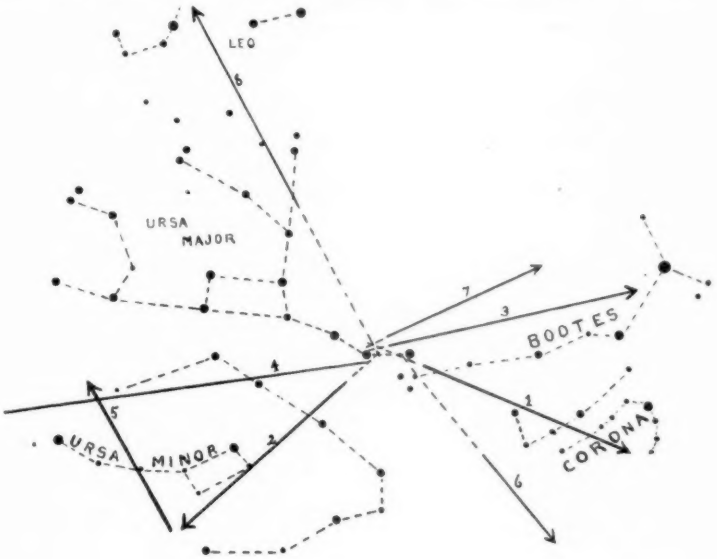
LAST CONTACT.

2^h	39^m	$34^s.9$	Twenty-inch	Herbert A. Howe.
2	39	32 .9	Six inch	Chas. J. Ling.
2	39	33 .9	Five-inch	Julian O. Howe.

The times predicted from the data in the American Ephemeris were respectively $0^h\ 44^m\ 25^s.1$ and $2^h\ 39^m\ 38^s.0$. HERBERT A HOWE.
University Park, Colo.

The ζ Ursids—On the night of the 27th of April I saw a meteor appear between ζ and η Ursae Majoris. On the 28th, I was looking at the same place as the night before, and I saw a meteor of the first magnitude. It came from the same place as the one the night before. After a little I saw another faint one, and then another; they all shot from the same place. I then made a rough map, and watched for more. I saw after this four more, besides two Serpentids, and one Lyrid. 1 was on the 27th, it was red. On the 28th, 2 was bright, medium-speeded, white meteor, which shot into the clouds below; 3 was of fifth magnitude, slow, faint and shot parallel to ζ and η Ursae Majoris; 4 was of the 5th magnitude, green; 5 was a Lyrid, of second magnitude, red; 6 was short, of a blue color, fourth magnitude; 7 was a white meteor, of third magnitude; 8 was in Leo, white, of second magnitude; 9 was near Leo, off map, third

white, and came from Serpens; 10 was a bright meteor. It started from Serpens, went under Corvus and disappeared below it. It was low in the south and had a curved track, left no trail, was of a pure white color and of 0 magnitude. It rivalled Venus in whiteness and brightness. The radiant is between ζ and η



METEORS FROM URSA MAJOR.

Radiant between ζ and η, April 27 and 28, 1900.

Time	Color.	Magn.	Class.	No.	Date and Remarks.
9.00	Red	2d	ζ. U. M.	1	April 27 exactly at 9.00
8.27	White	1st	ζ. U. M.	2	" 28 quite bright.
8.30	White	5th	ζ. U. M.	3	" 28 faint, slow.
8.35	Green	5th	ζ. U. M.	4	" 28 "
8.42	Red	2d	Lyrid	5	" 28 came from Lyra.
8.48	Blue	4th	ζ. U. M.	6	" 28 a blue meteor.
9.03	White	3d	ζ. U. M.	7	" 28
9.06	White	2d	ζ. U. M.	8	" 28 In Leo.
9.07	White	3d	Serpentid	9	" 28 near Leo.
9.10	White	0	Serpentid	10	" 28 beautiful bright meteor.

ζ. U. M. = Zeta Ursæ Majoris. Average = one in every 7½ minutes.

ROBERT M. DOYLE.

A letter has been received at the Harvard College Observatory from Professor H. A. Howe at Denver stating that Eros was observed with the 20-inch refractor of the Chamberlin Observatory with the following results:

Gr. M. T.	Apparent R. A. h m s	Apparent Decl. ° ′ ″	Comp. Stars.
May 27.90720	23 47 3.43	+ 2 46 27.3	Boss 8197
May 27.91859	23 47 4.37	+ 2 46 38.6	Boss 8198

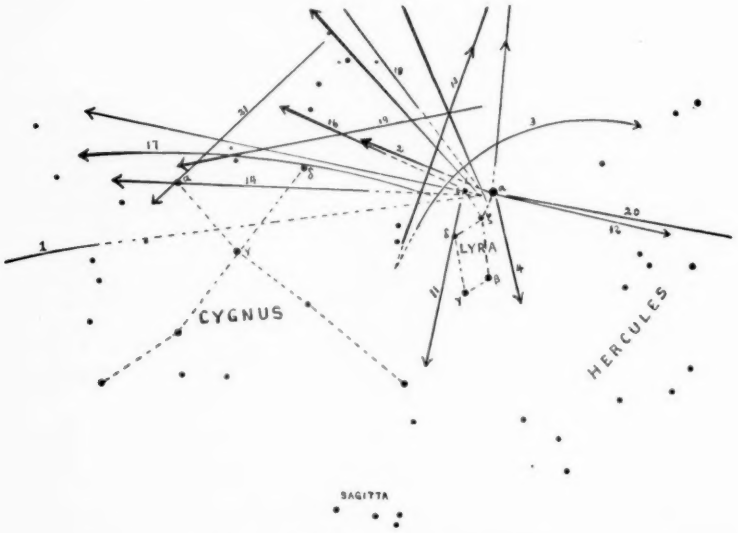
After taking parallax and aberration into account, a comparison of these observations with the ephemeris of J. B. Westhaver in A. J. No. 479 gave the following corrections to that ephemeris:

Gr. M. T.	s	″	
May 27.90729	+ 1.7	+ 28	Estimated Magn. 13.
May 27.91859	+ 1.5	+ 28	

So far as known this is the first visual observation of Eros since its conjunction with the Sun.

HARVARD COLLEGE OBSERVATORY, Cambridge, Mass.,
June 1, 1900.

The Lyrid Shower.—The nights of the 17th, 18th, 19th were cloudy, but the 20th was clear. The Moon was one day before last quarter. The radiant point was not very well defined but most of the meteors came from Vega, or between the region of α, ε, ζ Lyrae. Five of the meteors were outside of this triangle, numbers 13, 21, 3, 19 and 11. They were classed as Lyrids, because they were all very swift, excepting No. 3. At 12:30 my observations began, but one meteor was seen of third magnitude, medium speed, moving from Vega at 12:20. No. 2 was a short, swift and remarkably bright meteor, leaving a trail; 3 was an unusual meteor, it started slow and seemed to have several colors, it left a trail of red, white and blue colors. Another peculiarity of it was a very curved



THE LYRID METEORS, APRIL, 20, 1900.

path; 4, 5 and 6 were minor meteors; 7 was blue, of the fourth magnitude, and quite swift, showing the characteristic of the Lyrid shower; 8, a repetition of No. 7; 9 was a most surprising meteor. It was so short, so bright and so swift, the eye couldn't quite make out whether it curved or not. It could not have lasted one-fifth of a second; 10 and 11 were faint; 12 was rather slow and very faint, of the sixth magnitude; 13 was a most beautiful sight. It started a little northeast of Lyra, and shot across the sky, disappearing in Virgo, passing Arcturus and Corona. It was of a red color, slow, left a persistent trail, lasting two seconds. After this the following meteors were so swift the eye just caught them. The shower was, I think, at its maximum on the 19th but, although the meteors on the 20th were few, yet t' ey were swift, bright, interesting. They diminished in number as 4:00 o'clock approached, so I think the maximum must have been before the 20th. The shower might be called good, on account of the many interesting points about individual meteors.

No	Color.	Magn.	Time.	Class.	Notes.	Trails.
1	W.	3	12.20	L	Mostly off map.	———
2	W.	0.5	12.43	L	Very bright, trail	4th magn. W.
3	R. (W. B)	0 5	1.01	L	of several colors.	Long R. 2d mag.
4	W.	3	1.10	L	Short one.	———
5	W.	2 5	1.28	L	Off map	———
6	W.	1	1.30	L	———	———
7	B.	4	1 35	L	Swift.	
8	B.	4	1 38	L	"	
9	W.	0	1 43	L	Very swift & bri't.	Bright.
10	B.	5	1.50	L	———	———
11	B.	4	2 01	L	———	———
12	——	6	2 07	L	Very faint.	Across sky.
13	R W.	1	2.10	L	Beautiful.	2d magn. B.
14	R.	2	2 35	L	Swift.	———
15	R.	2	2 44	L	Very swift indeed.	
16	R.	2	2.48	L	" " "	
17	R.	4	2 55	L	" " "	
18	W.	1	2 56	L	A flash of white.	
19	W. (B.)	0.5	3.13	L	Medium speed.	Red.
20	R.	1.5	3.16	L	Swift.	3d magn.
21	W.	3	3.20	M	Short curved.	———
22	R.	2	3 45	L	Swift.	———
Apr. 5	B.	2.5	7 57	L	Medium speed, curved quite a little.	

1.5, 0.5, 2 5 mean, a little less than first magnitude, zero magnitude and second magnitude. Average one every nine minutes. Average magnitude 2 5 or a little less than second magnitude.

ROBERT M. DOLE,

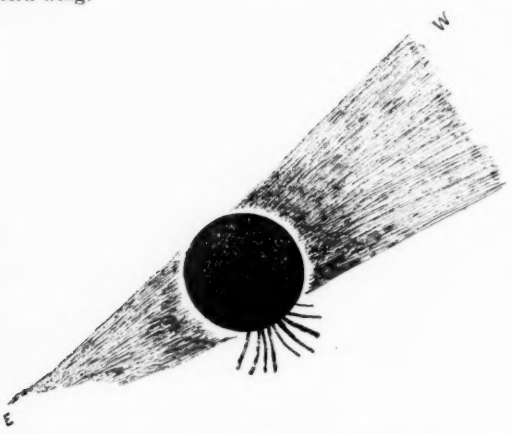
91 Glen Road, Jam. Plain, Mass.

April 21, 1900.

The Total Eclipse of the Sun at New Orleans.—The total eclipse was observed here in a remarkably clear area of a partly cloudy sky. The duration being scarcely more than a minute, the darkness was only that of early twilight. About ten seconds before totality the inner corona became visible.

The outer corona had two extensions one on the east tapering and not

sharply outlined on the edges, the other somewhat longer, being about a degree at its outer edge from the Moon's limb. It was more conspicuous, was sharply outlined, and widened as it extended. Its upper edge pointed almost directly towards the planet Mercury. It left an impression of a more streaky structure than the eastern wing.



CORONA OF TOTAL SOLAR ECLIPSE, MAY 28, 1900.
Seen at New Orleans, La., Drawn by Miss Rose O'Halloran.

The south polar rays were conspicuous pointed and curving.
The inner corona was shallow.
The tint of the entire appendage was that of feathering clouds after sunset.
May 28, 1900. ROSE O'HALLORAN.

Opposition of Eros in 1900.—The opposition of Eros during next autumn will afford opportunities for observations of especial interest. The near approach of the planet to the Earth will permit the solar parallax to be determined, while the great variations in phase and distance will give unusual value to photometric observations obtained at this time. The ephemeris of Dr. Millosevich, published in the Berlin Jahrbuch for 1902, provides the means for discussing the measures of position of Eros. The annexed table, which is based on this ephemeris, furnishes a part of the material required for the investigations mentioned above. The date is given in the first column. The right ascension and declination for 1855, for Berlin midnight, are given in the second and third columns. This epoch is selected for convenience in identifying Eros by comparison with the stars in the Durchmusterung. The daily motion in right ascension, expressed in seconds of time, and in minutes of arc when reduced to the equator, are given in the fourth and fifth columns. The daily motion in declination, and the total motion expressed in minutes of arc, are given in the sixth and seventh columns. These quantities are important in planning observations for parallax, especially those made photographically. The logarithm of the distance from the Sun has been kindly furnished by Dr. Millosevich and is given in the eighth column. The logarithm of the distance from the Earth is given in the ninth column. From this it appears that its distance when nearest the Earth is less than a third of that of the Sun from the Earth. This minimum occurs on December

26, nearly two months after opposition, which takes place on October 30. The phase angle between the Sun and Earth, as seen from Eros, is given in the tenth column. There are few asteroids for which this angle much exceeds 30°. In the table, beginning with the value 37°.5 it gradually diminishes to 28°.3 at about the time of opposition, and then gradually increases, until on January 31, it attains the extraordinary value of 56°.1, becoming even greater later. The photometric magnitude, neglecting the phase and assuming that the light is inversely proportional to the squares of the distances of the Earth and the Sun, is given in the eleventh column. It is based on the measures described in H. C. O. Circular No. 34, from which it appears that the magnitude would be 11.39 at a distance of unity from the Sun and Earth, and that the photographic magnitude is 0.6 fainter than the photometric. It will be noticed that these last values are nearly 0.8 fainter than those given by Dr. Millosevich, who based his magnitudes on visual observations. As the magnitude 9.5 in the Durchmusterung is about 10.5 on the photometric scale, this difference is readily explained. The difference becomes still greater if we apply a correction for phase. This correction, in the case of the asteroids, is about $0.03p$, in which p is the phase angle. If we assume that this law can be applied to Eros for angles as great as 56° we obtain the corrected magnitudes given in the twelfth column. The phase angle in the observations described in Circular No. 34 is 21°.2. The magnitude at distance unity, therefore, becomes $11.39 - 0.64 = 10.75$. The approximate mean time of meridian transit is given in the thirteenth column, and the aberration time in the fourteenth.

EPHEMERIS OF EROS.

Date. 1900-'01	R. A 1855.	Decl. 1855.	Daily Motion. R. A.	R. A.	Decl.	Tot.	log. r.	log. Δ.	Phase	Magn.	C. Magn.	M. Tr.	Aberr.
	h m	° ′	*s*	′	′	′			°			*h m*	*s*
Sept. 1	2 18.9	+ 33 27	+ 70	+ 15	+ 22	25	0.1830	9.9688	37.5	11.85	12.33	15 39	404
9	27.5	+ 36 25	+ 60	+ 12	+ 22	25	.1701	.8692	36.6	11.62	12.08	15 16	369
17	34.6	+ 39 20	+ 45	+ 9	+ 23	25	.1688	.8285	35.4	11.38	11.80	14 52	336
25	39.2	+ 42 29	+ 25	+ 5	+ 23	24	.1613	.7871	34.0	11.13	11.51	14 25	305
Oct. 3	40.8	+ 45 30	− 1	0	+ 22	22	.1535	.7456	32.5	10.89	11.23	13 55	278
11	38.7	+ 48 20	− 32	− 5	+ 20	21	.1455	.7048	31.0	10.64	10.93	13 21	253
19	32.0	+ 50 52	− 68	− 11	+ 17	20	.1374	.6654	29.6	10.40	10.65	12 43	231
27	20.6	+ 52 49	− 101	− 15	+ 12	19	.1291	.6286	28.6	10.18	10.40	12 0	212
Nov. 4	5.7	+ 53 55	− 121	− 18	+ 5	19	.1207	.5954	28.3	9.97	10.18	11 14	196
12	1 49.3	+ 54 0	− 120	− 18	− 3	18	.1124	.5666	29.1	9.79	10.02	10 26	184
20	35.0	+ 53 0	− 92	− 14	− 11	18	.1041	.5430	30.9	9.63	9.92	9 40	174
28	25.8	+ 51 2	− 45	− 7	− 18	19	.0961	.5248	33.0	9.49	9.86	8 59	167
Dec. 6	23.5	+ 48 22	+ 9	+ 2	− 22	22	.0884	.5117	36.8	9.39	9.85	8 25	162
14	28.2	+ 45 16	+ 60	+ 11	− 24	26	.0812	.5033	40.2	9.31	9.88	7 58	159
22	39.3	+ 41 56	+ 106	+ 20	− 26	33	.0746	.4994	43.7	9.26	9.93	7 38	157
30	56.0	+ 38 31	+ 143	+ 28	− 26	38	.0686	.4993	47.0	9.23	10.00	7 23	158
Jan. 7	2 17.1	+ 35 7	+ 172	+ 35	− 26	44	.0636	.5029	49.8	9.22	10.07	7 13	159
15	41.6	+ 31 43	+ 194	+ 41	− 25	48	.0595	.5101	52.2	9.24	10.17	7 6	161
23	3 8.6	+ 28 23	+ 210	+ 46	− 25	52	.0566	.5210	54.3	9.28	10.27	7 1	165
31	37.3	+ 25 7	+ 219	+ 50	− 24	55	.0548	.5353	56.1	9.34	10.38	6 58	171

As an example of the use of this table let us consider the most favorable conditions for determining the solar parallax. It soon appears that this problem is by no means a simple one. If we select the end of December, when Eros is nearest the Earth, we find that meridian transit occurs so early in the evening that Eros cannot be photographed far east of the meridian. Moreover, the motion both in right ascension and declination is so great that if the telescope is made to follow the stars, Eros will trail so rapidly over the plate that it may not leave any impression on it. If the total diurnal motion is 24′, the motion will be 1″ a minute. If, then, the diameter of the image is 2″, Eros cannot be photographed, unless an

exposure of two minutes is sufficient. In such a case it may be necessary to make the telescope follow on Eros and not on a star. All the stars will then appear as short trails which are easily bisected. If the motion of Eros is large, its position with relation to the comparison stars will differ greatly when east and when west of the meridian. Moreover, it will be necessary to measure the total motion and after subtracting the large motion of Eros determine the small remaining parallax. During the latter part of January Eros culminates at nearly the same time on successive nights and will thus be favorably situated for observations west of the meridian for several weeks. The path of Eros has a loop extending over about 13° in right ascension and 20′ in declination, and with a center at about R. A. $2^h\ 5^m$, Dec. +51°. The point of crossing is at R. A. $2^h\ 21^m.9$, Dec.+34°25′ (1855). It is, therefore, not far from the stars +34°447, magn. 9.3, and +34°448, magn. 8.0. Eros will pass through the point of crossing on September 3, 1900, and again on January 8, 1901. Photometric observations, if made on these dates, will have especial value, since the same comparison stars can be used for both.

A photograph of Eros was obtained on September 6, 1898, with the 11-inch Draper telescope, whose focal length is 153 inches. Stars of the ninth magnitude are readily photographed with this instrument in 5 seconds. The exposure was 10 minutes, the daily motion 18′, and the computed magnitude 12.1. Allowing for the difference in motion it would be equally difficult to photograph Eros on this date and on September 17, 1900.

EDWARD C. PICKERING

HARVARD COLLEGE OBSERVATORY Circular No. 49.
February 14, 1900.

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